



AGAINST KOLÁŘ'S CRITICISM OF TICHÝ'S BARE INDIVIDUALS

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Abstract: In the present paper it is defended Tichý's conception of bare individuals against criticism raised by Petr Kolář. Kolář's attempt was in fact misguided because in most of his definitions he construed bare individuals as individuals lacking (certain kind of) properties. However, from Tichý's repeated formulations it is clear enough that bare individuals are individuals such that for any contingent property the individual instantiates, it is possible to lack it. Thus in fact, Kolář criticized conception(s) which was completely not Tichý's own. We discuss also other reasons why Kolář's criticism is misguided.

The Czech philosopher and logician Petr Kolář tried to put into rigorous terms Tichý's formulations of antiessentialism, the conception of bare individuals. Kolář expressed his views in the three his publications, especially 'Individual Nudism' (Kolář 1999a) and 'Clothing Bare Individuals' (Kolář 2000), so the topic seems to be one of his most important intellectual themes. However, his attempt ends with dismissing Tichý's original idea. To defend Tichý's conception of bare individuals, which was extensively discussed by the present author mainly in (Raclavský 2008) and (Raclavský 2008a), I will show various reasons why Kolář's criticism of Tichý is mistaken. Basically, Kolář criticized a theory which was entirely not Tichý's own (or similar to it), therefore Kolář provided no real reason for repudiating of Tichý's views. To disclose how Kolář arrived to the final conclusion that the doctrine of bare individuals is wrong, I will concentrate on his most extensive contribution (Kolář 2000). Tichý as well as Kolář are working within the background of intensional logic, thus properties are construed as functions from possible worlds (conceivable alternative to the

actual world, sets of states-of-affairs; we will ignore temporal dependency) to classes of individuals which are extensions of these properties.¹

Tichý published his conception for the three times (*cf.* Tichý 2004, 717, Tichý 1983, 241, Tichý 1988, 210), in each case nearly by the same words. We can put the essential as follows:

an individual is bare a) not in the sense of actually lacking properties, but in the sense that b) where P is any non-trivial property the individual instantiates, c) it is metaphysically possible for it (or: it might conceivably) to lack P; and it still be the same thing without thereby becoming its own numerically distinct individual and only trivial properties the individual cannot possibly lack

Trivial properties (a term borrowed from Plantinga) are properties such that their extension is the same in every possible world; trivial properties are thus constant functions, their courses of values are stable, these properties are “trivial” (for the definition of trivial properties see the Appendix A).²

It should be stressed that in the part a) Tichý explicitly warn us before the conclusion that bare individuals are individuals without properties (i.e. ‘naked individuals’). In the part b) there is contained a word which we understand as the universal quantifier binding variable for properties that occurs also in the part c). Thus parts b) and c) are in fact general categorical statement of the form $\forall f((\dots f \dots) \rightarrow (\dots f \dots))$ (where f is a variable for properties). We will see that Kolář’s definitions usually change the direction of the implication (\leftarrow). One times he changed also the existential quantifier binding possible worlds (in Tichý’s formulation ‘is possible’, ‘is -able’ that we naturally understand as ‘there exists at least one possible world such’), from modal logic known as \diamond , into the universal one, in modal logic it is \square . The two heavy changes in the formulation of the conceptions of bare individuals have in fact fatal consequences. For it would be possible to show that there are properties the individuals must instantiate but definitions based on Kolář’s misconceptions do not allow them; so they will be rejected as insatisfiable, thus materially inadequate.

In order to support the unacceptability of the conception of bare individuals formulated the way suggested by Kolář we need the following concept of trivial property (Kolář 2000, 128; whenever we will mention Kolář’s sense of trivial, we indicate it by ‘K’ in superscript):

f is a *trivial*^K property =_{df} all individuals necessarily (i.e. in every possible world) have *f*

¹ Nearly all material of this paper is adopted from the manuscript written in May 2005.

² Take into consideration that the intuitive sense of ‘trivial’ concerns with the invariance in extension (that is its triviality). One can also find Tichý’s own phrase ‘constant (i.e. trivial) function’ (*cf.* Tichý 2004, 209, 758).

Kolář claimed that such definition of the property “trivial” is given ‘in a manner [Pavel Tichý] should like’ (*ibid.*, 129). We will see that this claim is wrong. For let us ask how many properties are trivial^K in Kolář’s sense. Any property f which has the same extension(s) on the respective argument(s) as another property g is identical with that property (the extensionality principle for properties; *cf.* Raclavský 2007). Hence, *there is only one trivial^K property* – the property which has as its unique non-varying extension the class of all individuals. We can call that property *the trivial universal property*, here is the definition:

being a *trivial universal* property $f =_{df}$ being a property such that in every possible world the extension of f is identical with the only class s which is the complement of the empty class

However, in his formulation of the doctrine of bare individuals Tichý repeatedly talked about trivial properties, not about one trivial property (*cf.* Tichý 2004, 717, Tichý 1988, 210). Possible confusion can be caused by the fact that there are plentitude of concepts (or: constructions³) of this special property that is individuated as a function from possible worlds. Among other concepts of the trivial universal property are those expressed by ‘being such individual that $2+3=5$ ’, ‘being such individual that humanity is humanity’, etc. However, the explication of properties we are managing in this paper is intensional, not hyperintensional. Therefore, self-identity, conceptually grasped by means of the concept defined as follows:

being a *self-identical* individual $x =_{df}$ being an individual such that x is identical with x is the property identical with the trivial universal property (due to the extensionality principle for properties). But the trivial universal property must be clearly distinguished from completely different trivial properties such as trivial properties having specific singleton as its non-varying extension – called them *trivial singular properties*. One instance:

being an individual *identical with I_1* $=_{df}$ being an individual x such that it is identical with I_1

Therefore, *Kolář’s explication of Tichý’s notion of trivial is entirely inadequate*.⁴ Hence, *Kolář’s use of trivial^K leads to an improbable interpretation of Tichý*. No surprise that from his basically mistaken definition of trivial properties Kolář easily derived his final appreciation of the doctrine of bare individuals:

the specification of the doctrine of bare individuals ... leads to nowhere (Kolář 2000, 135)

³ Constructions are structured entities, procedures, which construct set-theoretical objects like mappings, intensions included; constructions are hyperintensions. (See Tichý 1998, Tichý 2004.)

⁴ For example, Tichý (*cf.* Tichý 1988, 210) explicitly mentioned the property of “being numerically identical with Etna” (“Etna’s own numerical identity”), i.e. an example of a trivial singular property (not of the trivial universal property “being numerically self-identical”).

For it is easy to show that no individual, say I_j , can lack, for instance, the identity with I_j , the property that no other individual may instantiate. This makes the real core of Kolář's argumentation. Let us see it in details.

Kolář suggested just three definitions of “being bare” (a property of individuals), the third one in four variants.⁵ His first definition (Kolář 2000, 129) is just:

being bare^{K1} =_{df} having only trivial^K properties

As an explication of Tichý's doctrine of bare individuals it is, no doubt, quite futile for several reasons. For instance, the definiens has the form $\forall f ((f x) \rightarrow (\text{Trivial}^K f))$ and this corresponds to none Tichý's formulation. But Kolář realized only that individuals not capable to instantiate any non-trivial property are really dubious entities (*ibid.*, 130). Of course, we agree with him. However, it was sufficient for Kolář to claim that an individual I_k cannot be without identity with I_k . It is clear enough that the conception of bare individuals is not meant as a silly doctrine according to which individuals are ontologically without its own numerical identity.

Kolář's second definition is not substantially better (*ibid.*, 130):

being bare^{K2} =_{df} having only trivial^K properties necessarily

Kolář refuted this definition by means of the property “having the same height as Mick Jagger” (*ibid.*, 130) which is necessary for M. Jagger but not for any other individual. This property is called by Cmorej ‘partly essential property’ but Kolář forgot to follow his relevant papers. The extensions of partly essential property vary across the logical space, so “having the same height as Mick Jagger” is not trivial and it is not trivial^K too. Note again that any trivial singular property such as “being identical with I_k ” can disprove definition like K2. One may be tempted to suppose that if Kolář had made just this observation, he could change his insufficient definition of trivial properties. It may be added that the form of Kolář's K2, i.e. $\forall f (\Box(f x) \rightarrow (\text{Trivial}^K f))$, corresponds to no Tichý's formulation of the doctrine of bare individuals

Finally, Kolář suggested relatively plausible definition (*ibid.*, 133):

being bare^{K3} =_{df} can lack any contingent property

Kolář did not suggest formal pendant of K3, but if we follow his own formalizations of K1-K2, we get just $\forall f (\Diamond \sim(f x) \rightarrow (\text{Contingent } f))$ (supposing that $\sim(f x)$ is a formalization of ‘to lack a property’). If we change the direction of the implication, we would get – for the first

⁵ Kolář's own argumentation is complicated by asking whether this or that property of individuals he defined is trivial or not. But we set such examinations aside. Whether this or that property is trivial, in our sense, is indicated by vertical bars in formal expressions exposed in appendices.

time – something similar to Tichý’s formulation. Unfortunately, Kolář did not choose this way. Moreover, he used the term ‘contingent’ he did not define. That led him, perhaps, to the suggestion of the four following readings of this definition.

The *first reading*:

K3a) Any individual may lack any of its *empirical* properties yet not all of them in the same world.

K3a contains, surprisingly, the appendix ‘yet not all of them in the same world’ which seems to be superfluous when we compare it with the original definition of Bare^{K3}.⁶ Kolář did not comment this. Moreover, Kolář’s own argument against K3a) is quite strange. He told us that K3a) is a mere tautological definition ‘an individual may lack these properties which it may lack’; but he would like an informative, synthetic definition (*ibid.* 134). One must ask what the definition (in alleged tautological sense) is here. Because the claim K3a) is not a definition in the strict sense but a statement attributing to such-and-such individuals certain properties. The definition of Bare^{K3a} using (after a suitable modification) K3a) would be synthetic because it would introduce a ‘new’ concept (namely Bare^{K3a}) by means of other, more primitive, concepts. Kolář’s objection is thus invalid.

The *second reading* of Bare^{K3} uses the term ‘non-trivial^K’ (*ibid.*, 134), i.e. the complement of ‘trivial^K’:

K3b) Any individual may lack any of its *non-trivial^K* properties yet not all of them in the same world.

It was easy for Kolář to refute this definition because the (unique) extension of the property “non-trivial^K” involves, *inter alia*, trivial singular properties. Hence, it is obvious that individual I₁ cannot lack the trivial singular property “being a member of the set {I₁}” (*ibid.*).

The *third reading*:

K3c) Any individual may lack all of its *empirical* properties in the same world.

is not a viable one, says Kolář, because an individual lacking all empirical properties is ‘necessarily cognitively ungraspable’ (*ibid.*). I am convinced that any individual instantiates infinitely many empirical properties in any world (an individual lacking empirical properties is hardly an individual which are among object within our empirical framework).⁷

⁶ By empirical properties Kolář might mean properties purely empirical plus partly empirical (i.e. partly essential), or only purely empirical; but we do not know. S.c. partly essential properties (such as “having the same height as M. Jagger”) are not lackable by certain individuals what would exclude K3a) as a viable reading.

⁷ Cf. (Cmorej 2001), 107 (T5), (Cmorej 2006), 147-150; (Raclavský 2007a; published with a big delay), 16-17, (Raclavský 2007b), 7, (Raclavský 2008). In (Cmorej 2006) there are disputed Kolář’s definitions of bare individuals and Cmorej refuted them for the reason that any individual has plenty contingent properties in any

Nevertheless, I am obliged to say that Kolář's counter-argument is invalid for the following reason. The idea, that to cognitively grasp an individual by means of properties it instantiates is necessary for knowing whether the individual under inspection is the very same individual, leads to fatal epistemological circle. The possible world framework is designed as a framework concerned with testing of definite items, individuals, on the instantiation of properties. To perform successfully a test on an individual, say I_1 , on being a wooden, one must take – i.e. to grasp cognitively first – this I_1 and the test apply only after that. It is futile to choose an individual only by its being wooden and then test it on its being wooden. It would be also futile to go further and claim that only after such test we are sure that I_1 identical with itself. For we already know what I_1 is – it is that object identical with I_1 . To cognitively grasp I_1 amounts to cognitively grasp it as I_1 . (For more detail argumentation see Tichý 1983, section IV, from which I have adopted my claims.)

Finally, the *fourth reading*:

K3d) Any individual may lack all of its *non-trivial* properties in the same world. can be easily refuted by the same reason as K3b), which was noticed also by Kolář (*ibid.*, 134-135), i.e. by means of his unrealizable demand that bare individuals have only the trivial universal property necessarily. Therefore, “being identical with I_1 ”, which is a non-trivial^K property, is not dispensable for I_1 in any world, contrary to the postulation K3d).

A reader might have noticed that K3c)-K3d) has one significant feature: they were formulated as conditionals such that a quantifier binding properties was introduced in the antecedent, i.e. they share the form such as $\forall x ((\forall f \sim (f x)) \rightarrow (... f...))$. This amounts to consider possible world such that an individual lacks all properties (of certain kind) in that world, i.e. to construe individuals as *naked*. Kolář is not the only one who attacked bare individuals by the same erroneous understanding of the theory of bare individuals. If we turn to Tichý's formulation we may easily check that the universal quantifier binding properties is introduced not in the antecedent or in the consequent but before the whole such condition, i.e. his claim is of a form $\forall x \forall f ((... f...) \rightarrow \sim (f x))$. Clearly, the statement says that bare individuals are individuals such that for every property if it is such-and-such, it is possible that that individual lack it. Such claim does not state that there is a possible world in which individuals are without all properties of certain kind. Moreover, in (Raclavský 2008) I put also various arguments for the claim that any individual has in any possible world a plentitude of (purely) contingent properties, thus it is never naked, and that this fact is completely

possible world (I completely agree with this Cmorej's claim; various reasons for the adoption of such view see in Raclavský 2008a).

compatible with the conception of bare individuals, i.e. that it does not contradict to the very idea of bare individuals.

On the other hand, K3a)-K3b) do not construe individuals as without all properties of certain kind. In contrast to the reasons for their rejection suggested by Kolář, we may offer more persuasive arguments. K3b) is easily refutable by means of the reference to the singular trivial properties – an individual I_k cannot lack the property “being identical with I_k ”, a property which is non-trivial^K (we thus see the disastrous consequences of Kolář’s wrong definition of trivial properties again). K3a) can be rejected for the same reason as soon as we construe empirical properties as non-trivial^K properties; we are free to do it, since Kolář did not suggest what he meant by empirical properties. But consider otherwise: let the empirical properties are those properties Tichý called non-trivial. On this assumption, K3a) is still wrong. Suppose a property alternating the empty class and the singleton $\{I_2\}$ as its extensions. It is clear enough that this property is empirical but I_1 (as well as any other individual distinct from I_2) cannot lack it (on the natural understanding of ‘can lack’ an individual can lack something what it can, in certain possible world, have). Albeit K3a) is wrong, Tichý’s own formulation is not refutable by means of the same reason. Because in the part b) Tichý explicitly mentioned that the respective individual has this empirical property (when I_1 does not have the discussed empirical property, the antecedent formulated in b) is false, not true).

To sum up, Kolář thoroughly forbore to understand Tichý’s conception of bare individuals. Therefore, his criticism is simply misguided. This reason, supported by our above analyses, is enough in order to defend Tichý’s conception before Kolář’s objections.⁸ Last note: Kolář suggested in his papers another conception of bare individuals but it does not concern, really surprisingly, with individuals at all – we investigate it (and refute it) in the appendix C).⁹

⁸ Despite that, it can be shown that Tichý’s definition of bare individuals is, in fact, wrong. Nonetheless, it can be repaired such a way that it becomes to be real formulation of the conception of bare individuals; both were the target of (Raclavský 2008).

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¹⁰ Tichý's (sometimes co-authored) published papers are reprinted (if they were not originally published in English, they are translated into English) in (Tichý 2004).

Appendices

A) Formal definitions of concepts

In the shortcut notation of transparent intensional logic (see Tichý 1988, Tichý 2004) explained in (Raclavský 2007) (we omit mainly the signs for trivializations and indication of temporal dependence¹¹) we can offer formal definitions of various concepts mentioned in our text. (Definitions from appendix A) have already appeared in Raclavský 2007; see this text for more related details.) Tichý's logic is a higher-order intensional logic handling total and partial function, constructed by s.c. constructions, over collections of individuals (ι), truth-values (o ; comprising T and F), possible-worlds (ω) and real numbers/time-moments (τ). Variable x (or y) constructs ι -objects (individuals), variable w (or w' , w'') constructs ω -objects (possible worlds), variable s constructs $(o\iota)$ -objects (classes of individuals), variable f (or g) constructs $(o\iota)_\omega$ -objects (properties of individuals), variable o constructs o -objects, variable p constructs o_ω -objects (propositions). The type $(o\iota)_\omega$ will be written briefly as φ , the type o_ω as π . Compositions $[X w]$ will be written as X_w . The sense of each such definition is to specify which object would be constructed by the construction on the left side. Both constructions related by the operator \equiv^ξ construct, dependently on any valuation, the very same object (if they construct, with respect to particular valuation, anything at all). Constructions on the both sides are open constructions; for easier understanding we will indicate behind $'//'$ the missing binding string like $'\lambda w [\lambda x f'$, which may close each of the constructions. (If the constructions in the immediately further definition should be closed by the same binding string, we do not repeat this indication. Of course, the reader should complete in his mind the proper record of construction by the respective number of the right brackets on the right places.) In the definitions following some previous definitions, we will use η -reduced (even η -normalized) forms of constructions from the previous definitions (schematically, $\lambda xy [X xy]$ is η -reducible to X). The type ζ (let ζ or ζ_i be an arbitrary type) written in $'\equiv^\zeta'$ is a type of object constructed by the construction (on each side) after its closing by the respective binding item. Note however, that the equality \equiv^ζ does not relate just ζ -objects but certain ζ_i -objects which are constructed by open constructions on both sides; thus the type of \equiv^ζ is in fact $(o\zeta_i\zeta_i)$. Nevertheless, the inscription $'\equiv^\zeta'$ contains the information about which type the type ζ_i properly is – when ζ is, for instance, $(o(o\iota)\varphi)_\omega$ and we read $'// \lambda w [\lambda s f'$ behind the definition,

¹¹ The temporal versions of our definitions are easy to obtain. It would be sufficient just to write $'\lambda w \lambda t'$ instead of $'\lambda w'$ and $'..._{wt}...'$ instead of $'..._{w...}'$ (using here the convention that $[[X w] t]$ is abbreviated as X_{wt}). Of course, when it is used, for example, the variable w'' then we should use the variable t'' .

then ξ_i is $(o(oi)\phi)_\omega$ minus ω (due to ' λw [\cdot '] and minus $(oi)\phi$ (due to ' $\lambda x f$ '), thus ξ_i is just o (i.e. ' $\equiv^{(o(oi)\phi)\omega}$ ' denotes here an equality of type (ooo)). If not indicated, all defined intensions are total functions; if these intensions are trivial, we put the variable w (in the left part of the definition) into the vertical bars.

Few preliminary concepts (the definition of the 'totalizing' predicate 'true' is necessary for handling partiality of properties):

$$[\text{True}_w p] \equiv^{(o\pi)\omega} [\exists \lambda o [[o = p_w] \wedge [o = \top]]] \quad // \lambda w [\lambda p]$$

$$[\text{ExtensionOf}_w f] \equiv^{(oi)\phi\omega} [\text{sing.}\lambda s [s = f_w]] \text{ (i.e. } \equiv^{(oi)\phi\omega} f_w) \quad // \lambda w [\lambda f]$$

(The construction $[\text{ExtensionOf}_w f]$ may be improper, i.e. does not construct anything at all, if f is not defined in given particular w ; "extension of" is a partial mapping.)

Few kinds of properties of individuals (already known to Tichý):

$$[\text{Total}_{|w|} f] \equiv^{(o\phi)\omega} [\forall \lambda w' [\exists \lambda s [s = [\text{ExtensionOf}_{w'} f]]]]$$

$$[\text{Partial}_{|w|} f] \equiv^{(o\phi)\omega} [\exists \lambda w' \neg [\exists \lambda s [s = [\text{ExtensionOf}_{w'} f]]]]$$

$$[\text{Trivial}_{|w|} f] \equiv^{(o\phi)\omega} [\forall \lambda w' [\text{True}_{w'} [\lambda w'' [\text{ExtensionOf}_{w''} f] = [\text{ExtensionOf}_w f]]]]$$

$$\vee [\neg \exists \lambda w'' [\exists \lambda s [s = [\text{ExtensionOf}_{w''} f]]]]]$$

$$[\text{NonTrivial}_{|w|} f] \equiv^{(o\phi)\omega} \neg [\text{Trivial}_w f]$$

(When some property is total, then its correct complement (the property complementary to the former property) is constructible by means of negation placed in the former concept-construction.)

Definitions of two sorts of trivial properties of individuals:

$$[\text{Self-Identical}_{|w|} x] \equiv^{(oi)\omega} [x = x] \quad // \lambda w [\lambda x]$$

$$[\text{IdenticalWith}_{|w|} x I_1] \equiv^{(oi)\omega} [x = I_1]$$

Definitions of the trivial universal property(-ies), the trivial singular properties:

$$[\text{Universal}^{\text{Tr}}_{|w|} f] \equiv^{(o\phi)\omega} [\forall \lambda w' [[\text{ExtensionOf}_{w'} f] = [\lambda x \top]]] \quad // \lambda w [\lambda f]$$

(The construction $[\lambda x \top]$ constructs the class of all individuals.)

$$[\text{Singular}^{\text{Tr}}_{|w|} f] \equiv^{(o\phi)\omega} [[\forall \lambda w' [\exists \lambda x [[[\text{ExtensionOf}_{w'} f] x] \wedge [\forall \lambda y [[[\text{ExtensionOf}_{w'} f] y] \rightarrow [y=x]]]]]] \wedge [\text{Trivial}_{|w|} f]]]$$

B) Formalizations of Kolář's concepts

Adapting Kolář's own proposals (*ibid.*, 138):

$$[\text{Triv}^{\text{K}}_{|w|} f] \equiv^{(o\phi)\omega} [\forall \lambda x [\forall \lambda w' [f_w' x]]] \quad // \lambda w [\lambda f]$$

$$[\text{Bare}^{\text{K1}}_{|w|} x] \equiv^{(oi)\omega} [\forall \lambda f [[f_w x] \rightarrow [\text{Trivial}^{\text{K}}_{|w|} f]]] \quad // \lambda w [\lambda x]$$

$$[\text{Bare}^{\text{K2}}_{|w|} x] \equiv^{(oi)\omega} [\forall \lambda f [[\forall \lambda w' [f_w' x]] \rightarrow [\text{Trivial}^{\text{K}}_{|w|} f]]]$$

Additional remark: On p. 138 (*ibid.*) Kolář claimed that constructions constructing properties and also quantifying over properties are ‘ill-defined’ and cites one of Russell’s formulations of Vicious Circle Principle. It seems, however, that Kolář did not understand TIL’s ramified hierarchy of types at all. Every construction constructs (if it constructs) something other than itself. Therefore no variable constructing constructions is allowed to construct also itself. But variables constructing properties are innocent, thus there is nothing wrong with an ‘impredicative’ definition defining certain property by means of reference to all properties. Clearly, a property, as a mere mapping, is such that any structure or any part of concepts-construction of it is lost in it. It is then without harm when certain construction using with quantifier quantifying over mappings which do not contain constructions. Vicious Circle is repudiated within TIL by ‘stratification’ of constructions into orders, thus no construction say of order i can quantify (by the medium of variable) over constructions of the same order; a construction quantifying over i -order constructions is $i+1$ -order construction. Russell’s Vicious Circle Principle should be applied only when they are investigated Russell’s structured attributes (*cf.* Tichý 1988, 226, where Tichý talks about orders of attributes).

Our formalizations of Kolář’s concept Bare^{K3a} with respect to more primitive concepts:

$$\begin{aligned}
[\text{Lack}_w x f] &\equiv^{(\text{op})\omega} \neg[\text{True}_w [\lambda w' [f_w' x]]] && // \lambda w [\lambda x f] \\
[\text{CanLack}_{|w|} x f] &\equiv^{(\text{op})\omega} [[\exists \lambda w' [f_w' x]] \wedge [\exists \lambda w'' [\text{Lack}_{w''} x f]]] \\
[\text{Contingent}_{|w|} f] &\equiv^{(\text{op})\omega} [\text{NonTrivial}_w f] && // \lambda w [\lambda f] \\
[\text{Bare}^{\text{K3a}}_{|w|} x] &\equiv^{(\text{ot})\omega} [\forall \lambda f [[\text{CanLack}_w x f] \rightarrow [\text{Contingent}_w f]]] && // \lambda w [\lambda x]
\end{aligned}$$

C) Kolář’s bare individuals as individual offices

We have seen, that Kolář’s formulations of the property “being a bare individual” are based on the quite odd assumption that properties whose only extension (assigned to it in all possible worlds) is not the whole domain of individuals are not trivial. Consequences of this evident error led him to the revision of the concept of bare individual. But his own proposal is as follows (Kolář 2000, 136):

bare individual^{KU} =_{df} an individual office having a unique non-empty extension which is uniform across all possible worlds

We can put it into the definition:

$$\begin{aligned}
[\text{BareIndividual}^{\text{KU}}_{(w)} u] &\equiv^{(\text{ot}(\omega))\omega} [\forall \lambda w' [[\exists \lambda x [[\text{ExtensionOf}_w^u u]=x] \\
&\wedge [\text{ExtensionOf}_w^u u]=[\text{ExtensionOf}_w^u u]]]
\end{aligned}$$

where u constructs individual offices, ExtensionOf^n constructs world dependent mapping from individual offices to their holders.¹² Note that “being bare individual^{KU}” is not the property of individuals but the property of individual offices (it is an $(o \iota_\omega)_\omega$ -object). One cannot help to smell certain philosophical newspeak here. “Being a bare individual” is not a property of individuals but a property of individual offices, we are told. When somebody asks ‘What amounts for an individual to be bare?’, Tichý would say that to be an individual which is such-and-such. But Kolář seems to think that to give such natural answer is wrong and he suggests answering in the style of ‘to be such-and-such individual office’. It should be also noted that these offices are constructible by constructions of form $\lambda w I_j$. Such offices are occupied by particular individuals thus it is perfectly reasonable to ask then whether these individuals are bare or not. However, Kolář answers quite different question (*ibid.*):

An ordinary individual (=particular) is the extension of a bare individual in any world. But then he returns back and claims that the property “being bare individual^{KU}” is a trivial^K property of offices and he indicates that he solved Tichý’s original demands. But he is simply wrong because not every individual office has this property, thus it is not trivial^K. It is, however, a trivial property in our sense because those offices, which happens to instantiate it, have it invariably in all possible worlds thus the extension of “being bare individual^{KU}” is still the same in all worlds. The only benefit from Kolář’s proposal is the definition of trivial individual offices which are occupied. In other words, we need not to apply Occam’s razor, since Kolář did not define superfluous concept (this concept is identical with the concept TrivialSingular , suitably type-theoretically adopted, which occurs also in Raclavský 2007).

¹² This definition is in Kolář’s intention. He himself used a type-theoretically ill construction instead of it, *ibid.*, 138, for there is no identity relation between individuals and the empty set of individuals, \emptyset .