
# Type Indicators

Jiří Raclavský

**Abstract**: Due to recent suggestion of Pavel Materna (using Tichý's transparent intensional logic using hierarchy of types) there are expressions which only indicate logical type of some object and which are not expressive of concepts but only of "preconcepts" (hence they should not appear in analyses of sentences or co-expressions.) I rename these expressions to *type indicators of objects in chosen hierarchy of types* (so they are names of types over respective basis). I expanse the list of indicators discovered by Materna by addition of names of all types (of the respective hierarchy of types). Moreover, from this kind of indicators (which I call type indicators of object constructed by variable) I distinguish two other kinds of indicators (type indicators of object constructed by trivialization, type indicators of object denoted by used expression). Then I show expressions which are only seemingly type indicators. Type indicators can be viewed as "primitive terms" in every typed-λ-calculi-like artificial language.

## INTRODUCTION

Objects, we are aiming at, are often not of the same kind. The traditional task of ontology is to classify objects by certain criteria (for example, properties were found as distinct from bearers of these properties, etc.). The need for classification of objects in the frame of logic has arisen from the effort to prevent set theoretical paradoxes; Bertrand Russell is then known for the suggestion of theory of types for such purpose. I will not mention here development of the theory of types I will neither compare it with the categorial grammar which provides classification on the syntactical level. But I will discuss a specific phenomenon occurring when typed lambda calculus (developed by Alonzo Church) is used for the logical analysis of language. Specific apparatus (a bit modified typed lambda calculus) I choose for the demonstration of that phenomenon is transparent intensional logic (TIL for brief)

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originated by Pavel Tichý (see at least [Tichý 1988]). Note that my observations hold (with some special exceptions) also for another apparatus that use typed λ-calculi.

A co-developer of TIL Pavel Materna introduced in his last book (Conceptual Systems, [Materna 2004]) useful term helpful to logical analysis of natural language, s.c. *preconcept*. He considered preconcepts as being not concepts in a logical sense but as entities preceding a chosen conceptual system. In sequences of words expressions for preconcepts indicate logical type (i.e. belonging to the type of the definite hierarchy of type) of object connected with some expression. It is important to notice that expressions for preconcepts are not concepts and thus they are not included in the analysis of investigated sentence or co-expression (being not concepts, they cannot be parts of (sentence) meaning which can be viewed as a certain composition of component concepts).[1] It is a certain exception from application of Parmenides principle (see [Duží, Materna 2002]) according to which the logical analysis must cover just those entities that are expressed by subexpressions of given sentence.[2]

The choice of epistemic basis (collection of atomic types) precedes any hierarchy of types. Materna enumerated as preconcepts just (and only) names of atomic types ([Materna 2004], p. 80). However, it is very natural to consider preconcepts as being the names of all types of chosen hierarchy of types (over chosen epistemic basis).[3] Let me thus allow here to rename preconcepts to *type indicators* (*of chosen hierarchy*). Generally speaking, type indicators are *primitive* (i.e. nonanalyzable) *terms* for kinds-types of entities of a given ontology.

The aim of this study is an extension of preconcepts mentioned by Materna. I have classified them into three groups with respect to the ways they indicate. Then I will show expressions which seemingly behave as type indicators, but in fact they are not type indicators (with regards to chosen hierarchy of types). The whole study may be understood as a contribution to the analysis of sentences frequently used by theoreticians applying TIL when they comment their analyses of (common) natural language.[4]

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[1] Naturally, under the condition that the conceptual system to which we relate our logical analyses is fine-grained enough to distinguish subconcepts expressed by given language.
[2] I modified their proposal a bit, but it is not important to discuss it here.
[3] Concepts of some chosen conceptual system serve for conceptual identification of objects classified by certain hierarchy of types.
[4] This text (originally written in 2005) is dedicated to my teacher and colleague Pavel Materna on the occasion of his 75-th birthday.

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## BASIC DEFINITIONS OF TIL APPARATUS[5]

The key definition determines s.c. constructions (more precisely: kinds of their forming) which are considered as "structured meanings", "procedures" (as meanings they are objective, abstract entities). The definition originates from typed lambda calculus understood purely objectual way what leads to certain modifications. The lowliest objects are typically reached by *trivializations* (marked by '$^0$') or by *variables* (like *x*, *y*); variables are not conceived as letters but as constructions constructing various objects on the basis of valuation. Application called *composition* serves for an application of a constructed function onto an argument; frequently they are used compositions of the form [[X *w*]*t*] (in the notation shortened to $X_{wt}$). Lambda abstraction is called in TIL *closure*; frequently they are used constructions of functions from possible worlds and times, λ*w*[λ*t* [...*wt*...]] (abbreviated to λ*w*λ*t* [...*wt*...]). TIL incorporates a specific *ramified theory types* (it is used also in the definition of constructions) – among *atomic types* there are individuals (ι), truth values (ο), possible worlds (ω) and time moments/real numbers (τ); from those they are generated *molecular types* such as of classes, functions, relations. *Intensions* are functions from possible worlds and time moments, objects generally of type ((ατ)ω), in abbreviation $\alpha_{\tau\omega}$ (where α is an arbitrary type). Constructions constructing objects of these types belong to the higher order types (type of constructions is generally $*_n$).

### Definition of types – types of order 1

Let B be a basis. As a chosen consider *an epistemic basis B* consisting of pair-wise disjoint classes of individuals (ι), truth values (ο), real numbers / time moments (τ), possible worlds (ω).[6]

*1) Types of order 1 (T)*

($t_1$1) Every member of B is a *type of order* 1 *over* B.

($t_1$2) If 0<*m* and $\alpha,\beta_1, \ldots, \beta_m$ are *types of order* 1 over B then the collection ($\alpha\beta_1 \ldots \beta_m$) of all *m*-ary (total and partial) mappings from $\beta_1, \ldots, \beta_m$ into $\alpha$ is also *type of order* 1 *over* B.

($t_1$3) Nothing is a *type of order* 1 *over* B unless it so follows from ($t_1$1) and ($t_1$2).

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[5] Detailly commented and exhaustive presentment of TIL can be gathered, for example, from [Tichý 1988], [Materna 2004].

[6] Remark: The hierarchy of types chosen by us (which is often used in TIL) is built over chosen epistemic basis B. Over such basis it can be built another hierarchy of types (which, for example, can contain the type of ordered *n*-tuples).

**Definition of constructions**

*1) Variable (x).* The *variables* $x_k$ construct the *k*-th entity from the given sequence of entities yielded by the *valuation*. For any valuation *v*, the variable *v-constructs* what the valuation *v* assigns to it; so it is always *v-proper*. Variables are *open* constructions because they construct entities dependently on valuation.

*2) Trivialization* ($^0X$). Let *X* be any entity (object or construction). *Trivialization* is the construction of which the starting point, as well as the outcome, is *X* itself, $^0X$ constructs *X* without any change. What is *v*-constructed by $^0X$ never depends on *v*, so $^0X$ is *proper* for any *v*.

*3) Composition ([X $X_1$...$X_n$]).* Let *X* be a construction that *v*-constructs a mapping *M* of type ($\alpha\beta_1...\beta_n$) and let $X_1$, ..., $X_n$ are constructions that *v*-construct entities $E_1$, …, $E_n$ of types $\beta_1$, ..., $\beta_n$, respectively. If the mapping *M* is defined on $<E_1, ..., E_n>$, then the construction [*X* $X_1...X_n$], the *composition of* constructions *X*, $X_1$, ..., $X_n$ (in this order), *v*-constructs the value (an object or a construction) of *M* on that *n*-tuple. If *X* does not *v*-construct a mapping which is defined at the *n*-tuple of entities *v*-constructed by $X_1$, ..., $X_n$, then it is *v-improper*.

*4) Closure ([$\lambda x_1...x_n$ Y]).* Let $x_1$, ..., $x_n$ be arbitrary pairwise distinct variables *v*-constructing entities of types $\beta_1$, ..., $\beta_n$, respectively, and *Y* a construction *v*-constructing members of a type $\alpha$. For any valuation *v*, the *closure* [$\lambda\, x_1 \ldots x_n\ Y$] *v*-constructs the mapping *M* which takes any $<E_1, ..., E_n>$ (where $E_1$, ..., $E_n$ of the respective types $\beta_1$, ..., $\beta_n$) into that member (if any) of $\alpha$ which is $v(E_1/x_1, ..., E_n/x_n)$-constructed by *Y*, where valuation $v(E_1/x_1, ..., E_n/x_n)$ is like *v* except for assigning $E_1$ to $x_1$, ..., and $E_n$ to $x_n$. For any *v*, the construction [$\lambda\, x_1 \ldots x_n\ Y$] is *v-proper*.

**Definition of hierarchy of types – construction of order *n* and types of order *n*+1**

*2) Construction of order n* ($C_n$)

($c_n$ 1) Let $\xi$ be *any type of order n over* B. Every variable ranging over $\xi$ is a *construction of order n over* B. If A is of (i.e., belongs to) type $\xi$ then $^0$A is a *construction of order n over* B.

($c_n$ 2) If $0<m$ and $X_0$, $X_1$, ..., $X_m$ are *constructions of order n then* [$X_0$ $X_1 \ldots X_m$] is a *construction of order n over* B.

($c_n$ 3) If $0<m$, $\xi$ is a *type of order n over* B, and Y as well as the distinct variables $x_1$, …, $x_m$ are *constructions of order n over* B, then [$\lambda x_1 \ldots x_m$ Y] is a *construction of order n over* B.

($c_n$ 4) Nothing is a *construction of order n over* B unless it so follows from ($c_n$ 1)-($c_n$ 3).

*3) Types of order n+1 ($T_{n+1}$).* Let $*_n$ be the collection of *all constructions of order n over* B. The collection of *types of order n+1 over* B is defined as follows:

($t_{n+1}$ 1) $*_n$ and every *type of order n* is a *type of order n*+1.

($t_{n+1}$ 2) If 0<*m* and $\alpha,\beta_1, \ldots, \beta_m$ *types of order n+1 over* B then the collection $(\alpha,\beta_1, \ldots, \beta_m)$ of all *m*-ary (total and partial) mappings from $\beta_1, \ldots, \beta_m$ into $\alpha$ is also a *type of order n+1 over* B.

($t_{n+1}$ 3) Nothing is a *type of order n*+1 *over* B unless it so follows from ($t_{n+1}$1) and ($t_{n+1}$ 2).

Remark: *Concept is α-, η-normalized closed construction*.[7]

**Denoting scheme**

In TIL it is usually used the following (Church's like) denoting scheme:

expression

*expression expresses:*

meaning (construction)

*expression denotes, construction constructs:*

denotatum (extension / intension / other construction / nothing)

Whereas mathematical or logical expressions do not refer dependently on modal or temporal parameter, in the case of other expressions their reference typically varies, we call them empirical expressions. TIL strictly distinguishes these two kinds of expressions. *An empirical expression* is an expression which denotes a nontrivial intension.[8] The value of denoted intension, i.e. *a referent* of the expression in certain world and time cannot be find out by logic; for example, the empirical sentence 'The number of planets (of our solar system) is 8' has in actual world and time the truth-value true but it may obtain another truth-value (if any) in other worlds and times. Intensions are partial mappings; they can be undefined at some arguments. On the other hand, non-empirical expressions (typically logical or mathematical expressions) denote extensions, i.e. such objects whose modal and temporal invariability is left, thus nonempirical expression are then understood as not denoting a trivial mapping from possible worlds and times, but simply the respective value.[9]

## KINDS OF TYPE INDICATORS

[7] For more see [Materna 2004].
[8] Nontrivial intensions have different values for at least two arguments.
[9] But in the case of need we can consider them as trivial (constant) intensions; trivial intensions have the same value for all arguments.

**A) Type indicators of object constructed by a tacit variable**

First of all let us consider the sentence:

Some individuals are interesting.

Just in the case of such sentences Materna realized that it is surely superfluous to attribute to the values of the respective variable (the variable constructs individuals) that they are individuals (and then to attribute to them that they are interesting) what is usually done when we use Russell's style of analysis of positive existential judgment:

$\exists x$ (Individual($x$)$\wedge$Interesting($x$))

The expression 'individual' serves here only for determining the logical type of objects constructed by the variable. Thus this expression does not express a concept, it only indicates type.[10] Thus the analysis should be as follows:

$\lambda w \lambda t$ [$^0\exists^\iota$ [$\lambda x$ [$^0$Interesting$^\iota_{wt}$ $x$]]]

(The variable $x$ constructs objects of type $\iota$, $\exists^\iota$ is the respective quantifier (a class of classes of individuals, $(o(o\iota))$, the expression 'be interesting$^\iota$' denotes here an empirical property of individuals, an object of type $(o\iota)_{\tau\omega}$, the variable $w$ constructs objects of type $\omega$, the variable $t$ constructs objects of type $\tau$ (all variables are objects of order $*_1$, respectively constructions of order $*_2$).) From the analysis it is easily seen that it does not contain (a construction of) conjunction which was not named in the original sentence – thus the analysis aptly fills the principle of aboutness ("Parmenides principle"), the principle about what the expressions are.

In contradistinction to the opinion of Prof. Materna I am convinced that preconcepts-*type indicators indicate not only atomic types* (compare [Materna 2004], p. 80) *but all types over chosen epistemic basis*, that is also molecular types, types of constructions or types in which figure constructions (for example, functions from individuals to constructions etc.). Hence, for example, the sentence:[11]

Some (propositions) are true.

in which stands – as a type indicator – the expression 'proposition' has the logical analysis:

$\lambda w \lambda t$ [$^0\exists^{(o\tau)\omega}$ [$\lambda p$ [$^0$True$_{wt}$ $p$]]]

where the variable $p$ constructs objects of type $o_{\tau\omega}$, i.e. propositions (functions from possible worlds and times to truth values), $\exists^{(o\tau)\omega}$ is the respective quantifier (classes of classes of

[10] Of course, in common language the expression 'individual' has several connotations (as well as the expression 'proposition') which collides with logical terminology; I will ignore these usual connotations as it is common in logic.

[11] Putting type indicators in round brackets; it is also recommendable for praxis of analyticians.

propositions, $(o(oo_{\tau\omega}))$, 'True' denotes an empirical property of propositions, $(oo_{\tau\omega})_{\tau\omega}$. Again, in the analysis they are not inserted type indicators, nor conjunction (which is not named in the sentence).[12] In the analogous way we would analyze the sentence:

Some (constructions) are interesting.

The variable *c* constructs a construction (for the sake of simplicity we will consider only constructions of the first order), $\exists^{*1}$ is the respective quantifier $(o(o*_1))$, 'to be interesting*' denotes here an empirical property of constructions, an object of type $(o*_1)_{\tau\omega}$:

$\lambda w \lambda t\ [{}^0\exists^{*1}\ [\lambda c\ [{}^0\mathrm{Interesting}*_{wt}\ c]]]$

Note also one interesting thing: a variable which constructs objects of type which is not indicated is not explicitly shown on the sentential level – it must be discovered by the logical analysis; this variable serves (in a construction of analyzed expression) for constructing of some object (on the level of expressions we can speak about a tacit use of a variable). We can call this kind of type indicators "*type indicators of object constructed by a tacit variable*".[13]

Let us add that they are other positions where the type indicator can appear. For example, it can possibly stand in the end of sentence (the analysis is similar to our first example):

${}^0$Xenia constructs (an individual).

The analysis ('to construct' denotes a relation between constructions and individuals, an object of type $(o\ *_1\ \iota)$, '${}^0$Xenia' is – as mentioned construction – introduced by trivialization; compare below the type indicators of kind B):

$[{}^0\exists^{\iota}\ [\lambda x\ [{}^0\mathrm{Construct}\ {}^{00}\mathrm{Xenia}\ x]]]$

Of course, one can find sentences, whose author did not fit adequately correlative combination of types. For example, in the sentence:

Some (numbers) are true.

the property of proposition is attributed to numbers but such property of propositions is not type-correlative with numbers. This phenomenon is known as "a category mistake".[14] (Except

[12] One can object: "In η-reduced form, $\lambda w \lambda t\ [{}^0\exists^{(o\tau)\omega}\ [{}^0\mathrm{True}_{wt}]]$, the expression 'proposition' is not represented at all"; well, but in η-expanded it is. What is more, many constructions do not have η-reduced forms of such shape, for example, the sentence 'Some (individual) loves all (individuals)' is analyzed as: $\lambda w \lambda t\ [{}^0\exists^{\iota}\ [\lambda x\ [{}^0\forall^{\iota}\ [\lambda y\ [{}^0\mathrm{Love}_{wt}\ x\ y]]]]]$ (let 'Love' denotes an empirical relation between individuals) which is not further η-reducible and the type indicators are evidently represented.

[13] Another apparent example of this kind of type indicator indicating other kind of construction has not been discovered until now.

[14] In his classification of nonsenses Petr Kuchyňka called such nonsenses 'type nonsenses' ([Kuchyňka 1999], p. 895).

this kind it is possible to construe such co-expressions or sentences according to which certain object of some type is related to the object of type which is not a member of chosen hierarchy of types over specific basis; I will come back to this topic in the section "Seeming type indicators".)

**B) Type indicators of object constructed by trivialization**

In the previous section we have seen that from the logical point of view some words of common language are considered to be indicators of something what is constituted by logical analysis (this and the following kind of indicators are discoverable in language when it is investigated by logic). In sublanguages of mathematicians, logicians and philosophers they not rarely appear sentences which contain expressions functioning (from the view point of TIL) as indicators. A typical example of mentioning of a variable is in the sentence like:[15]

(The construction) *x* is interesting.

Again it is redundant to put in the analysis the construction of conjunction and the construction of predicate 'being a construction'; the expression 'construction' here serves only as y type indicator.

This kind of type indicators has certain connection with Materna's distinction *use / mention of concepts* (for example, 'A horse is a an ungulate' / 'The concept of horse is a zoological concept') in the frame of his theory of concepts (see [Materna 1998], pp. 102-106). However, mentioned constructions, like variables, are not concepts for Materna, because they are open constructions. Because there is a lot of more complicated constructions which are not concepts in Materna's sense, we can generally claim that *the mentioning of concepts falls under the mentioning of constructions*. Moreover, to be a concept is a nonempirical property of constructions (it is an object of type $(o*_n)$).[16] From this also follows that *the expression 'concept' is not a type indicator of object-construction constructed by trivialization* (nor any other kind of type indicator over selected hierarchy of types).

From Materna's theory about mentioning / using of concepts we can adopt a principle of analyses of such sentences. For example, in the analysis of sentence:

(The construction) *x* is interesting.

[15] Below I will not insert into sentence expressions for quantification ('some' etc.) or demonstratives (such as 'this') etc.

[16] The case of nonempirical properties is a bit simplified here – these are trivial intensions always assigning the same class; the modal as well as the temporal parameter may be thus ignored, dealing only with that constantly assigned class. The similar may be done also for nonempirical sentences (they denote trivial proposition). Such simplification should not affect the understanding of this article.

the mentioned variable will be introduced by trivialization. The expression ‘to be interesting’ denotes here an empirical property of constructions (let us restrict to the first order, $(o*_1)_{\tau\omega}$), the analysis is thus:

$\lambda w \lambda t\, [{}^0\text{Interesting}*_{wt}\ {}^0x]$

where the construction ${}^0x$ constructs an object of type $*_1$, i.e. the construction $x$.[17]

*In sentences not only variables but all kinds of constructions can be mentioned*. It can be *trivialization*:

(The construction of) Yannis is interesting.

with the analysis:

$\lambda w \lambda t\, [{}^0\text{Interesting}^{\iota}_{wt}\ {}^0[{}^0\text{Yannis}]]$

*composition*, for example

(The construction) $[{}^0\surd\ {}^0 25]$ constructs some (number).

with the analysis[18]:

$[\exists^{\tau}\, [\lambda t\, [{}^0\text{Construct}\ {}^0[{}^0\surd\ {}^0 25]\ t]]]$

where ‘to construct’ denotes a relation between constructions and numbers, $(o*_1\tau)$, the variable $t$ constructs numbers, $\exists^{\tau}$ is the respective quantifier, $(o(o\tau))$; or *closure*:

The construction $\lambda w \lambda t\, [{}^0\text{Interesting}*_{wt}\ {}^{00}\text{Yannis}]$ is interesting.

whereas the analysis of this sentence is:

$\lambda w \lambda t\, [{}^0\text{Interesting}*_{wt}\ {}^0[\lambda w \lambda t\, [{}^0\text{Interesting}*_{wt}\ {}^{00}\text{Yannis}]]\ ]$

Let us call this kind of indicators *type indicators of object constructed by trivialization*.[19] Well, according to terminology of TIL only constructions can be mentioned; thus the name *type indicators of mentioned constructions* can be also chosen.

Note that only the expression ‘construction’ is a type indicator of this kind. For type indication they do not serve expressions like ‘variable’, ‘trivialization’, ‘composition’, ‘closure’ because these are not types (or names of types) in the hierarchy of types used by us (for more, compare section “Seeming type indicators”).

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[17] Somebody would think that it is not a construction what is mentioned, she would think that it is the record of this construction. However, this would mean that the construction expressed by the quoted expression would be a construction of gödelian-code number. But in a given sentence the property “being interesting” is attributed to the construction not to certain number. Compare the end of C) and the end of section “Some problematical examples” for this problem.

[18] ‘Number’ is a type indicator of denoted object (i.e. number) which belongs to the kind of type indicators from the previous section.

[19] To insist that the expression mentioned in the sentence, for example ‘F’ (the expression denoting certain function), is a construction, is a deep error when disclosing type indication – the construction ${}^0$F is a construction, not the function. (Of course, the type of F can be indicated, but for this serves type indicators from the section C), see below.)

Now let us see examples of some other stylistic variants for this kind of type indicators. When observing the following sentence:

$[^0\div\ x\ ^00]$ is an improper (construction).

one can think that the property "be an improper construction", or "to be a construction", is attributed to that construction, hence that the expression "construction" is really not a type indicator of type of object constructed by trivialization. However, from the synonymous paraphrase:

(The construction) $[^0\div\ x\ ^00]$ is improper.

it is evident that 'construction' really is a type indicator of object constructed by trivialization. The analysis of both sentences is identical:

$[^0\text{Improper}\ ^0[^0\div\ x\ ^00]\ ]$

('be improper' denotes a nonempirical property of constructions, more precisely a class of constructions,[20] i.e. an object of type $(o*_1)$).

**C) Type indicators of object denoted by used expression**

Except already investigated mentioning of constructions or tacitly used variables we can find such sentences or co-expressions in which (again) the type of something is indicated. In this cases the type indicator is (usually) followed by an expression which is explicitly used, i.e. it is used for the direct denotation of certain object which is (typically) not a construction. For example, in the sentence:

$^0$Xenia constructs (an individual) Xenia.

the expression 'individual' is such a kind of type indicators which indicate type of object that is denoted by the expression following the type indicator. Hence the optimal name for such kind of indicators should be "*type indicators of type of object denoted by used expression*" (what can be shortened to "*type indicator of used expression denotatum*"). Type indicators of this kind are (similarly to the cases of two preceding kinds of type indicators) type indicators over given (chosen) epistemic basis. Thus, again, we do not insert into final analyses the possible analyses of such indicators. Hence the analysis of the given sentence is (the construction $^0$Xenia mentioned in the sentence is constructed by the trivialization):[21]

$[^0\text{Construct}\ ^{00}\text{Xenia}\ ^0\text{Xenia}]$

[20] This is an innocent "dualism".
[21] It was not indicated in the sentence by the type indicator of object constructed by trivialization, but it could be: "(The construction) $^0$Xenia constructs (an individual) Xenia".

These type indicators do not indicate only a type of proper name denotata, but generally denotata of all expressions (of certain language). In the case of sentences like:

(The property) to be a dog is interesting.

which can be rectified to the form:

(The property of individuals) canity is interesting.

which sounds a bit artificial (in other cases it is not so artificial, for example '(A property of individuals) vitreosity is interesting'). The expression 'to be interesting$^{\varphi}$' denotes a property of properties of individuals, an object of type $(o(o\iota)_{\tau\omega})_{\tau\omega}$, the expression 'to be a dog (i.e. canity)' denotes a property of individuals, an object of type $(o\iota)_{\tau\omega}$), in the analysis:

$\lambda w \lambda t$ [$^0$Interesting$^{\varphi}_{wt}$ $^0$Dog]

the construction of a property is in the supposition de dicto.

Similarly to the type indicators from the previous section B), we can find sentences, in which the expression for type indicating is in a less typical position (i.e. not before the expression whose type of denotatum it indicates):

Xenia is an interesting (individual).

Compare it with the other possibilities:

(The individual) Xenia is interesting.

Xenia, (an individual), is interesting.

(The individual) Xenia is an interesting (individual).

The analysis ('be interesting$^{\iota}$' again denotes an empirical property of individuals, an object of type $(o\iota)_{\tau\omega}$) is just:

$\lambda w \lambda t$ [$^0$Be_interesting$^{\iota}_{wt}$ $^0$Xenia]

Somebody would think about a separate kind of type indicators which she calls "type indicators of object denoted by mentioned expression"; when she meets sentences like:

Xenia is named 'Xenia'.

Note, however, that mentioned expression does amount to nothing else than the use of a name of this expression. These type indicators thus fall under type indicators of object denoted by used expression.

How do we analyze such sentences? Cited expressions of language can be represented by code numbers which are assigned to them by chosen gödelisation (these numbers are natural numbers).[22] In the respective construction we will represent the expression 'Xenia' by

[22] Such an attempt was provided within TIL by Pavel Tichý (compare at least [Tichý 1988], p. 228).

$^0$g(Xenia) where "g(Xenia)" should be certain gödelian number of given expression.[23] The analysis of the whole sentence is:

$\lambda w \lambda t$ [$^0$Be_named$_{wt}$ $^0$Xenia $^0$g(‚Xenia')]

where 'to be named' denotes an empirical relation between an individual and code number (an object of type $(o\ \iota\ \tau)_{\tau\omega}$). Another example:

' 'Xenia' ' is an expression for 'Xenia'.

for which there is the following analysis:

[$^0$Be_expression_for $^0$g('Xenia') $^0$g(Xenia)]

where 'to be an expression for' is (suppose) a nonempirical relation between two code numbers, an object of type $(o\tau\tau)$.[24]

**Suggestion for abbreviating convention of type indicator names**

During my personal experience I have found a tendency to abbreviate terms for type indicators because these precise terms are rather long. Let me present here such a convention for abbreviation (but I have to worry about inaccuracies of such shortened term below).

A. The term "type indicators of object constructed by tacit variable" can be shortened to "*type indicators of variable*" (or "type indicators of a tacit variable").

B. The term "type indicators of object constructed by trivialization" can be shortened to "*type indicators of trivialization*" (or "type indicators of a mentioned construction").

C. The term "type indicators of object denoted by used expression" can be shortened to "*type indicators of expression denotatum*".

The shortened term due to A. is inaccurate for the fact that variable can be also mentioned (as we discussed in section B). The shortened term due to B. is inaccurate for the fact that not only trivialization but a construction of any kind can be mentioned (however, the term "indicators of mentioned construction" is accurate enough). Finally, the shortened term due to C. is inaccurate for indicators of type A. (like 'individual') have its own denotatum (an object of type $\iota$ is indicated); similarly, type indicators of kind B. are related to mentioned (not used) expressions.[25]

Realize that all three kinds of type indicators have something common: they are concerned with denotation of certain expression.

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[23] It depends on the chosen method of gödelization. However, we do not suggest here any such particular method, thus we use "general inscription" only.

[24] Of course, such sentences should contain the message in which language (code) the first expression should be a name of the letter expression (etc.). I will ignore this also in the further examples.

[25] However, in the following text I will not use this abbreviating convention yet.

## SOME PROBLEMATICAL EXAMPLES

Let the first example of problematical case be a sentence which is frequently written as "Xenia has a property 'to be a human'" or "Xenia has a property "to be a human"":

Xenia has the property /of individuals/ being a human.

There are several ways how one can understand this sentence. However, we will refute three of them as not suitable.

a) According to the first reading of the given sentence is understood in the sense of:

Xenia has (an individual-property) for which there is an expression 'to be a human'.

where the expression 'individual property' is a type indicator of object constructed by a tacit variable. The analysis is:

$\lambda w \lambda t\, [{}^0\exists^{((o\iota)\tau)\omega}\, [\lambda f\, [{}^0\wedge\, [f_{wt}\ {}^0\text{Xenia}]\, [{}^0\text{Be\_an\_expression\_for}\ f\ {}^0\text{g('to be a human')}]\, ]]]$

(the variable $f$ constructs objects of type $(o\iota)_{\tau\omega}$, an empirical property of individuals, $\exists^{((o\iota)\tau)\omega}$ is the respective quantifier.) This reading is refutable for its ill-founded insertion of 'to be an expression', existential quantifier, and conjunction.

b) A bit similar reading in the sense of sentence:

Xenia has (an individual property) which constructs (a construction) to be a human.

which has an analysis ('human' denotes an empirical property of individuals, an object of type $(o\iota)_{\tau\omega}$ construction of which is introduced by trivialization, 'to construct' denotes a nonempirical relation between a property and a construction of second order, i.e. an object of type $(o\ (o\iota)_{\tau\omega}\ {}^*{}_2)$):

$\lambda w \lambda t\, [{}^0\exists^{((o\iota)\tau)\omega}\, [\lambda f\, [{}^0\wedge\, [f_{wt}\ {}^0\text{Xenia}]\, [{}^0\text{Construct}\ f\ {}^0[\lambda w \lambda t\, [{}^0\text{Human}_{wt}]]]\, ]]]$

we can refute for the similar reasons as the case a).

c) Another reading is in the sense of the following sentence:

Xenia instantiates (a property of individuals) being human.

where 'to instantiate' denotes an empirical relation between Xenia and a property, an object of type $(o\ \iota\ (o\iota)_{\tau\omega})_{\tau\omega}$, (the construction of this property is in the supposition de dicto):

$\lambda w \lambda t\, [{}^0\text{Instantiate}_{tw}\ {}^0\text{Xenia}\ {}^0\text{Human}]$

however, this reading also adds something extra – the expression 'instantiate'.

d) The most acceptable reading is based on ignoring these frequently used quotation marks, i.e. on the dismissing seeming indicating that "to be a human" is a cited expression or a mentioned construction. Disputed sentence thus simply expresses a construction:

$\lambda w\lambda t\,[{}^0\text{Human}_{wt}\ {}^0\text{Xenia}]$

thus the expression ‘property of individuals’ is a type indicator of object denoted by mentioned expression. Justification for such a reading can be seen from the passive form of this sentence:[26]

(The property of individuals) to be a human is such that Xenia has it.

Here is another example of a sentence which is at the first sight senseless:

The proposition that Xenia thinks is true.

and which is frequently written as “The proposition ‘Xenia thinks’ is true” or “The proposition “Xenia thinks” is true“. This sentence can be read also in several ways. However:

a) the reading in the sense:

There exists (a proposition) which is true and ‘Xenia thinks‘ is an expression for it.

in which the expression ‘proposition’ is a type indicator of an object constructed by variable,

b) as well as the reading in the sense:

There exists (a proposition), which is true
and which is constructed by (the construction) Xenia thinks.

can be both rejected for the similar reasons as we discussed in the case of problematical sentence above.

c) Thus we choose the following reading as the most adequate:

(The proposition that) Xenia thinks is true.

where the expression ‘proposition’ is a type indicator of mentioned expression denotatum; the analysis is the following construction:

$\lambda w\lambda t\,[{}^0\text{True}_{wt}\,[\lambda w\lambda t\,[{}^0\text{Thinks}_{wt}\ {}^0\text{Xenia}]]\,]$

(‘to think’ denotes an empirical property of individuals, an object of type $(o\iota)_{\tau\omega}$).

When we observe the following sentence we find certain surprising phenomenon:

The individual *x* is human.

If we immediately omit the reading in the sense that ‘*x*’ is an expression for certain individual or that ‘*x*’ serves here for constructing of certain individual (as we discussed in the first example), one can suppose that expression ‘individual’ is a type indicator of object denoted by mentioned expression, i.e. that ‘*x*’ is a mentioned expression. Thus its seeming analysis would be the construction:

$\lambda w\lambda t\,[{}^0\exists^{\iota}\,[\lambda x\,[{}^0\text{Human}_{wt}\ x]]]$

[26] With the analysis: $\lambda w\lambda t\,[\,[\lambda w\lambda t\,[\lambda x\,[{}^0\text{Human}_{wt}\ x]]]_{wt}\ {}^0\text{Xenia}]$ (where *x* is a variable constructing individuals).

But such an attempt of analyzing is refutable – to presuppose that '*x*' expresses '*x*' is an elementary error (in every case any expression expresses something distinct from itself). Also the following attempt of analysis is indefensible:

$\lambda w \lambda t\, [{}^0\mathrm{Human}_{wt}\ {}^0x]$

According to this "analysis" being a human is attributed to the construction-variable (that is constructed by trivialization), not to an individual.[27] For the explanation of this phenomenon compare our example with the sentence:

(The individual) $^0$Xenia is human.

whose seeming analysis would be the construction:

$\lambda w \lambda t\, [{}^0\mathrm{Human}_{wt}\ {}^0\mathrm{Xenia}]$

Here it is used a nonsensical interpretation that the expression '$^0$Xenia' expresses itself (i.e. '$^0$Xenia'). To construe the analysis in such a way that the construction $^0$Xenia is mentioned leads to the introducing of it by trivialization:

$\lambda w \lambda t\, [{}^0\mathrm{Human}_{wt}\ {}^{00}\mathrm{Xenia}]$

However, in such a case we would obtain "a category mistake" – no construction can be a human. Well, to state that in some context '$^0$Xenia' expresses '$^0$Xenia' but in some contexts it expresses '$^{00}$Xenia' is a hard analytical contextualism which is undesirable.

Thus we must state the fact which is obvious only within hyperintensional logical analysis by TIL: expressions for constructions may sometimes be introduced in the supposition – let us call it *suppositio conceptualis* – in which they express certain meanings, namely trivializations of these respective constructions. However, these expressions for constructions cannot be introduced as meaningful in supposition in which they express their "usual" meaning, i.e. one cannot use the expressions for constructions in *suppositio usualis* to express any meaning as it can be done for standard expressions – in our language expressions of constructions in suppositio usualis simply have no meaning.[28] To see it more clearly, note that expression like '$\lambda w \lambda t\, [{}^0\mathrm{Human}_{wt}\ {}^0\mathrm{Xenia}]$' understood as a sentence (i.e. the whole record of construction is in the suppositio usualis) is simply nonsensical in English (analogously for '/An individual/ $^0$X is a human' in which '$^0$X' stands in suppositio usualis, hence it is

[27] A construction like a variable can be surely interesting, but for expressing such a fact in the sentence one must use the type indicator of mentioned construction.

[28] Note that suppositio usualis / suppositio conceptualis is not the same as use / mention of expressions. When mentioning an expression, the quoted expression is introduced by the quotation marks, but the respective construction of such whole is a construction of gödelian number of the quoted expression; however, suppositio conceptualis means that the record of construction express a trivialization of the construction whose record is on the sentential level; but the record of construction in suppositio usualis express no construction because it is a meaningless string of signs. The theme of suppositio usualis / suppositio conceptualis is in greater details investigated in [Raclavský 2006].

meaningless subexpression), consequently nobody can put forth its logical analysis (i.e. construction).[29]

## SEEMING TYPE INDICATORS

An interesting group of expressions which cannot be considered as type indicators of objects over chosen hierarchy of types is comprised of such *expressions which are not names of type in the frame of chosen hierarchy of types*. Typical examples are expressions denoting kinds of constructions which are not classified by hierarchy of types used by us. For example, from our viewpoint the sentence:

The trivialization $^0$X is interesting.

says the same as sentence:

(The construction) $^0$X, which is a trivialization, is interesting.

To the (mentioned) construction $^0$X it is attributed here the nonempirical property 'to be a trivialization' (which is an object of type $(o*_1)$) and the empirical property 'to be interesting*' (an object of type $(o*_1)_{\tau\omega}$). The analysis is:

$\lambda w \lambda t$ [$^0\wedge$ [$^0$Trivialization $^{00}$X] [$^0$Interesting*$_{wt}$ $^{00}$X] ]

Similarly, sentences like:

Some compositions are improper.

which can be aptly read in the sense:

There exist (constructions) which are compositions and which are improper.

have analyses as the following one (the expression 'construction' is a type indicator of object constructed by variable, the expression 'to be a composition' is a nonempirical property of constructions, i.e. an object of type $(o*_1)$ (similarly for the expression 'be improper'):

[$^0\exists^{*1}$ [$\lambda c$ [$^0\wedge$ [$^0$Composition $c$] [$^0$Improper $c$] ]]]

The second specific subgroup of pseudo-indicators is made of examples of nonsenses which seemingly indicate (or: predicate)[30] type category which is not only a member of the hierarchy

[29] Of course, the sentence like 'There exists a value of (a construction) $x$, which is a human' is meaningful because the construction of variable is mentioned in the sentence. The expression 'construction' is the type indicator of object constructed by trivialization, thus the analysis is ('to be value of' denotes relation between value of variable and construction, an object of type $(o\iota*_1)$): $\lambda w \lambda t$ [$^0\exists^\iota$ [$\lambda x$ [$^0\wedge$ [$^0$Be_value $x$ $^0x$] [$^0$Interesting$^\iota_{wt}$ $x$] ]]]. This sentence is not synonymous with sentence like 'There exists (an individual) $x$ which is a human'.

[30] In this case it is a s.c. "type nonsense" in Kuchyňka's sense.

of types considered above, but also it cannot be detected the type of such expression-predicate denotatum. For example:

The ordered quadruple M is interesting.

The denotatum of the expression 'the ordered quadruple' cannot be sorted into whatever type in the hierarchy of types we consider. All such kinds of seemingly indicating expressions *are rightfully type indicators with respect to hierarchy of types which includes such specific types* for kinds of constructions, type of *n*-tuples, etc.

Another interesting group of seeming type indicators consists of *expressions which usually serve for the type indication but in some contexts they do not*. We can meet sentences like:

Xenia is an individual.

To consider here the expression 'individual' as a type indicator is wrong: when omitting the expression 'individual' it remains only 'Xenia is' what surely is not the whole information contained in the sentence. In the sentence 'Xenia is an individual' certain (of course, nonempirical) property is attributed to Xenia. It seems to me that it is wrong a) to give up analyzing such sentences, or b) to propose analyses like 'There exists (an individual) which is identical with Xenia'. With respect to a) it can be objected that in the sentences like 'Xenia thinks that Yannis is an individual' we would be completely lost in analysis of the target of Xenia's thinking. In the case b) we would insert to analysis an identity and existential quantifier what is unjustified because these entities are really not named in the original sentence.

Of course, one can think about "type nonsenses" such as 'Xenia is a number' (let us presuppose that 'Xenia' is not a name of any number but a proper name of certain individual). A speaker claiming such sentence makes "a category mistake". However, this does not change the fact that she is attributing certain (nonempirical) property to certain object; thus such sentence should be analyzed as having meaning: we really should analyze sentences like 'Yannis thinks that Xenia is a number' which can be true, thus it must be meaningful in the first place.

Hence *some expressions serve not only for indicating of a type ("to be of type"), but also for predicating a type ("to be a member of type"*). Such attributing of a type is surely not graspable within our chosen hierarchy of types (from this viewpoint the respective sentences are senseless). It is graspable within a hierarchy of types which has another dimension, thus types and classes of types can be categorized (such "two-dimensional" hierarchy of types was

designed by Marie Duží; see for example her [Duží 1999]). This enriched hierarchy of types is necessary for explication of meaning of sentences disputed in this paragraph.

In the end it can be mentioned that for the expansion of hierarchy of types we used during our study we would be provoked also if we would like to capture the meanings of sentences as:

The entity Kiliniki is interesting.

or:

The intension F is interesting.

etc., where the type seems to be indicated "generally". When trying to apply our hierarchy of types we found such sentences ambivalent – we are not able to determine of which type the object Kiliniki is or of which type precisely the intension F is.[31]

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Jiří Raclavský, PhD.
*Department of Philosophy*
*Faculty of Arts*
*Masaryk University*
*Arne Nováka 1*
602 00 Brno
Czech Rep.
e-mail: raclavsk@phil.muni.cz

[31] Are there other specific type indicators except the kind of construction? If we would not consider indication with respect to kinds of constructions used by us above, we can meet examples of indicating constructions of kind of single or double execution (construction of shape $^1$X, $^2$X), as in the sentence 'The result of (double executing of) $^0\sqrt{4}$ is 2" (its analysis is [$^0$Result_of $^2[^0\sqrt{4}]$ $^0$2], 'to be a result of' denotes an object of type (οττ)). Of course, such topic is also related to the theory of conceptual systems.