Duží's *De Dicto* and *De Re*: A Critical Review

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The theory of *de dicto* / *de re* is among topics which are still investigated by the theoreticians discussing the natural language. Among scholars using transparent intensional logic (TIL, for brief) Pavel Tichý, the originator of TIL, introduced his theory of *de dicto* and *de re* in "*De dicto* and *de re*" ([Tichý 1978]); in some sense final version of his theory one can find in his "Foundations of Frege's Logic" ([Tichý 1988]).[1] Another TIL scholar who was interested in *de dicto* / *de re* is Marie Duží whose theory of *de dicto* / *de re* is continually presented in her many articles from 2000 until now, especially [Duží 2000], [Duží 2001a], [Duží 2001b], [Duží 2003b], [Duží 2004]. Duží's text "Intensional Logic and the Irreducible Contrast between *de dicto* and *de re*" ([Duží 2004]) seems to be her most complex study on the topic because she presented there her theory in full extent together with many other claims related to *de dicto* / *de re*. Duží's theory is generally understood to be an advance on the original Tichý's theory of *de dicto* / *de re* and her other claims related to *de dicto* / *de re* are many times referred in her (or: her and Materna's) articles on other topics. During my studies in attitude logic I have met some unclear or queer places in her *de dicto* / *de re* which became to be more suspicious to me when I rediscovered and extended Tichý's theory of *de dicto de re*.[2] The present text is thus a critical review of Duží's *de dicto* and *de re*.[3]

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[1] However, for the first time Tichý speaks about *de dicto* / *de re* in his "An Approach to Intensional Analysis" ([Tichý 1971]).

[2] [Raclavský 2006].

[3] The apparatus of TIL can be found in [Tichý 1988], some extension of it in [Materna 2004]. Tichý's key concept, viz. *constructions*, are suggested by him to be *analyses* (or: *meanings*) of natural language expressions. The calculus of constructions arose from typed λ-calculi which is understood in an objectual way (application is

THE DEFINITION OF *DE DICTO* / *DE RE* SUPPOSITIONS

In her study "Intensional Logic and the Irreducible Contrast between *de dicto* and *de re*" Duží responded her previous work on *de dicto* / *de re* ([Duží 2000], [Duží 2001], [Duží 2001a]) in which she characterized these suppositions nearly by the same words as follows:

> *De dicto*: An expression E is in the *de dicto* supposition in the sentence S iff the truth-value of the proposition denoted by sentence S in a world *w* and time *t* is not determined (merely) by the value of the intension denoted by E in *that particular w, t* (but it is determined by the whole function). In other words, the intension is only *mentioned* (*dictum*) and is not *used* for obtaining its value. ... *De re*: On the other hand, we speak about the *de re* supposition when the reference of E / the value (*res*) of the denoted function "comes into play" as well: The truth-value of the proposition denoted by S (in *w, t*) depends on the value of the denoted function (in *that particular w, t*). ([Duží 2004], p. 13)

First of all, one would suggest improving of this formulation by "An empirical expression E …" and by the completing of the end of the last sentence by words "[of the denoted function] constructed by E"; one can then continue by suggestion to understand "*w*" and "*t*" not to be variables (a kind of constructions) but "w" and "t", i.e. particular values for "*w*" and "*t*".[4]

When one tries to understand this characterization of *de dicto* / *de re* suppositions, she may ask whether such subexpression of sentence S as it is the sentence S itself (let us mark this subexpression S') stands also in the supposition *de dicto* / *de re* in S. In which sense the truth-value of the proposition denoted by S (in the respective w, t) "depends on" the value of the function denoted by S'? The truth-value of proposition denoted by S in w, t is completely the same as the truth-value of proposition (in w, t) denoted by S' which is the subexpression of S. Because it seems that these values are really dependent (the truth-values of function denoted by S must be the same as the truth-values of function denoted by S') one is tempted

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called composition, λ-abstraction is called closure; Tichý works rather with s.c. trivializations of objects than with constants). In the basis of hierarchy of types, within TIL, there are recognized (as atomic objects) individuals (ι), truth-values (o), possible worlds (ω), time-moments (τ); empirical expressions denote (non-trivial) intensions, functions from possible worlds and time-moments (constructions are of special type, $*_n$). Expressions *express* constructions; expressions *denote* intensions / extensions which the respective constructions construct (that what is *referred to* by empirical expressions are values of intensions on the respective possible worlds, time moments).

[4] Somebody may be a bit surprised by terms "mentioned intension", "used intension" (further she talks about mentioned / used function) which were not precisely defined by her; but it is not important here.

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to say that S' is in the supposition *de re* in S. In this sense I understand the first part of Duží's own remark:

> The above preliminary characterization could serve almost as a definition, but according to it the sentence S itself would be in the *de re* supposition in S, which is not correct. The sentence "speaks about" (denotes) the whole dictum – the proposition, never its reference (res) – the truth-value in the actual world / time. ([Duží 2004], p. 13)

This leads Duží herself to repudiate her preliminary characterization of de dicto / de re.

However, I see one more important disadvantage of her "preliminary characterization": it is limited to indication of suppositions of expressions which are subexpressions only of sentences. Does the description "the Pope" stand in the supposition *de re* or *de dicto* in itself?, does "the Pope" stand in the term "the office of the Pope" in the supposition *de re* or *de dicto*?

Now let us investigate her "final" definition of *de dicto* / *de re* suppositions of which sense it is supposed to be clearer:

> Definition 7 (*De dicto / de re*) Let P be a propositional construction, i.e. P is of the form [λwλt X], where X constructs an object of the type o, and let us call the world / time couple constructed by *w, t* the *reporter's perspective.* Let C be an intensional construction occurring as a sub-construction within P. We say that C is a *de re constituent of* P if there is an intensional descent of the intension constructed by C to the reporter's perspective, i.e., if C is composed with ('applied to') these (reporter's) *w, t,* or if there is a propositional construction P', β_i-equivalent to P, in which the intension constructed by C is intensionally descended to the reporter's perspective. Otherwise C is *de dicto constituent of* P. ([Duží 2004], p. 15)

What is very surprising – *Duží did not define suppositions* (of anything) *at all*, she speaks only about *de re* / *de dicto* constituents. Also observe that there is another unclarity of her definition – she is not writing here about expressions (as she has written before); unfortunately, it can be found that *the relation between suppositions of constructions and suppositions of expressions remain entirely unexplained in Duží's 2004 (as well as in any other papers).*[5]

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[5] Cf. that Tichý has explained this relationship (see [Tichý 1988], p. 214).

Let us be charitable and presuppose that being a *de re* / *de dicto* component of some construction C means that this component is in the supposition *de re* / *de dicto* in C (realize that if we would not presuppose this, Duží's definition 7 is simply worthless).

In contrast to the preliminary characterization, here all "*w-*'s", "*t-*'s" are constructions-variables. The "reporter's perspective" is note made by these variables but by the world / time couple constructed by them, i.e. <w, t>. Duží's main claim is that C is in *de re* supposition (due to our consideration that she defined suppositions of constructions) in P if there is an intensional descent of the intension constructed by C to the reporter's perspective, i.e. to a couple <w, t> (let us sign this theory *IDR*). On the other hand, variables *w, t* are called by her "being reporter's" and the supposition *de re* of some construction C is defined as a composition of C with those variables (let us sign this theory of *de re CDR*). One can see that *such a definition is at least redundant* – one of CDR) or IDR) can be (or should be) omitted.

Definitions such a Definition 7 usually serves for deciding relevant cases. What is interesting for us here is the recognition whether some subconstruction S (constructing an intension I) of a construction C (which constructs a proposition P) is in the supposition *de dicto* or *de re* in C; we will show that for such purpose Duží's definition is generally useless. First consider some candidates for the analysis of the sentence:

(D) Xenia believes that the Pope is in danger.

Duží claims that this sentence is ambivalent and it should be disambiguated in the sense of the following constructions which Duží repeatedly published ('believe' denotes empirical relation between an agent and proposition, $(o\iota o_{\tau\omega})_{\tau\omega}$-object, 'be in danger' a property of individuals, $(o\iota)_{\tau\omega}$-object, 'the Pope' an individual office, $\iota_{\tau\omega}$-object; *w* constructs ω-objects, *t* constructs τ-objects):

($D^{*\text{dicto}}$) $\lambda w \lambda t \, [{}^0\text{Believe}_{wt} \, {}^0X \, [\lambda w \lambda t \, [{}^0DA_{wt} \, {}^0P_{wt}]] \,]$

($D^{*\text{re}}$) $\lambda w \lambda t \, [\, [\lambda x \, [{}^0\text{Believe}_{wt} \, {}^0X \, [\lambda w \lambda t \, [{}^0DA_{wt}x]]]] \, {}^0P_{wt}]$

According to Duží's definition (and both CDR and IDR) we can state that ($D^{*\text{dicto}}$) is *de dicto* in ($D^{*\text{dicto}}$), and ($D^{*\text{re}}$) is *de dicto* in ($D^{*\text{re}}$). We can also find that $[\lambda w \lambda t \, [{}^0DA_{wt} \, {}^0P_{wt}]]$ is *de dicto* in ($D^{*\text{dicto}}$), $[\lambda w \lambda t \, [{}^0DA_{wt}x]]]]$ is *de dicto* in ($D^{*\text{re}}$), ${}^0P$ is *de re* in ($D^{*\text{re}}$), ^{0}Believe is *de re* in ($D^{*\text{dicto}}$) and ($D^{*\text{re}}$). Well, *according to the Duží's definition we cannot solve whether the*

constructions of individuals as well as the construction-variables here, generally any construction of extension is in the supposition de dicto or de re.[6]

But there are other subconstructions which must interest us: ^{0}DA in (D*^{dicto}) and (D*^{re}), ^{0}P in (D*^{dicto}). *With help of CDR we may conclude that ^{0}P is also de re in (D*^{dicto}) and ^{0}DA is de re in (D*^{dicto}) and (D*^{re}) because they are composed with variables w and t.* But this is a counter-intuitive result – for example, for Tichý these constructions are simply in *de dicto* suppositions. Duží's formulation of CDR is thus dubious.[7]

Well, in her texts one can trace up a possible remedy improving CDR: α-renaming of variables *w* and *t* to *w'* and *t'*:

(D*'^{dicto}) $\lambda w \lambda t$ [^{0}Believe$_{wt}$ ^{0}X [$\lambda w \lambda t'$ [^{0}DA$_{w't'}$ ^{0}P$_{w't}$]]]

(D*'^{re}) $\lambda w \lambda t$ [[λx [^{0}Believe$_{wt}$ ^{0}X [$\lambda w \lambda t'$ [^{0}DA$_{w't'}$x]]]] ^{0}P$_{wt}$]

However, *Duží never related this "α-trick" to her definition of suppositions in any sense.* Moreover, this trick must be explicitly formulated: for example, it must be stated that $\lambda w' \lambda t'$ [...$_{w't'}$... [$\lambda w \lambda t$ [...$_{wt}$...]]...] is not a proper α-conversion (as being "reporter's" there are marked here variables of an agent); $\lambda w \lambda t$ [...$_{wt}$... [$\lambda w' \lambda t'$ [...$_{w't'}$...[$\lambda w \lambda t$ [...$_{wt}$...]]]...] is also not a suitable α-conversion; the variables of the wholly embedded construction must be also renamed to being not reporter's. Independently of this, suppositions of subconstructions ^{0}DA, ^{0}P in a construction like $\lambda w \lambda t$ [...^{0}[^{0}DA$_{wt}$ ^{0}P$_{w't'}$]...] would be decidable questionable way: ^{0}DA would be in the supposition *de re* (being composed with w, t of an agent) but ^{0}P would be in the supposition *de dicto* despite the fact that all these variables are ^{0}-bound thus both ^{0}DA, ^{0}P are rather *de dicto* in the whole construction. I conclude that *CDR is a valueless part of Duží's definition of de dicto / de re*.

Let us return to IDR. But IDR is worse than CDR. First ask: to which w, t couple is applied the intension constructed by ^{0}DA? Of course: to that couple one obtains by a valuation for *w* and *t* (i.e. <w, t>). However, by the definition of constructions, closures like those embedded in (D*^{dicto}) and (D*^{re}) ignore any partial valuation for *w* and *t* and dictate to run all possible valuations for *w* and *t*. Thus ^{0}DA is *de re* in both constructions (D*^{dicto}) and (D*^{re}) and the same holds for ^{0}P; thus there is no difference of their in suppositions between (D*^{dicto}) and (D*^{re}). *The incapability of real indicating of supposition makes IDR valueless.*

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[6] Btw. Tichý has dealt with the suppositions of such constructions ([Tichý 1988], pp. 213-214).

[7] Of course, a charitable reader may suggest: "Duží would like to say something like 'D is *de re* in C if it is composed with w, t which are free in C or λ-bound by outer lambdas of C'" (compare for example with [Tichý 1971], p. 286); I agree, but why Duží called her Definition 7 a definition and not only a 'characterization'?

But Duží's definition of *de dicto* / *de re* suppositions has also other important disadvantages: Generally, *no subconstruction of construction other than propositional construction (expressed by some sentence) is decidable by her definition as having subconstructions in supposition de dicto or de re* (is ^{0}DA *de dicto* or *de re* in non-propositional construction like $[^0D_{wt}]$?, and so on). Also *she in fact cannot handle with her* (presupposed) *definition of suppositions of constructions like ^{0}Actual, ^{0}Now constructing "semi-intensions"* (of types ω_ω, τ_τ; there are many other "semi"-intensions). One unexplained *part of her definition which Duží explained nowhere is her claim* "or if there is a propositional construction P', β_i-equivalent to P, in which the intension constructed by C is intensionally descended to the reporter's perspective.". To sum up, we have seen that *Duží's attempt to put a definition of de dicto / de re suppositions of expressions (or constructions) is entirely insufficient.* However, it must be stated that in his book Tichý did not show entirely clear and systematic criteria for recognizing of suppositions (btw. this does not change the fact that his theory is wider than that of Duží as we will see below).

Constructions of an existential quantifier and *de dicto* / *de re*

In the case of constructions with existential quantifier Duží claimed:

when comparing the use of an existential quantifier $\exists$ and the use of the property of existence E, we will speak about the "*de dicto*" / "*de re*" occurrence of $\exists$ according to the above characterization [i.e. the preliminary characterization of *de dicto* / *de re*; J.R.]

([Duží 2004], p. 13)

In the section 6) Duží explained what she had in mind:

(O1') $\lambda w\lambda t\ [^0B_{wt}\ ^0O\ [\lambda w^*\lambda t^*\ ^0\exists x\ [^0S_{w^*t^*}\ x]]]$ ^0B – *de re*, ^0S – *de dicto*, $^0\exists$ - "*de dicto*"[8]

(O3') $\lambda w\lambda t\ ^0\exists x\ [^0BOS_{wt}\ x]$ ^0BOS – *de re*, $^0\exists$ - "*de re*" [9]

Using our general characterization of the *de dicto* / *de re* distinction (the beginning of Section 4 [i.e. the preliminary characterization of *de dicto* / *de re*; J.R.]), we can say that the same distinction is displayed by the $^0\exists$ *component: "de dicto" in (O1') – the truth-value of the proposition constructed by (O1') in w, t – reporter's perspective – does not depend on the value of $^0\exists$, Ortcutt can believe that someone is a spy even if there are none, whereas in (O3')*

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[8] It is an analysis of the sentence "(O)rtcutt (B)elieves that someone is a (S)py".

[9] This is a shortened analysis of the sentence "Someone is (B)elieved by (O)rtcutt to be a (S)py".

the truth-value of the constructed proposition does depend on the value of $^0\exists$ - "de re", $\exists$ is

used.   ([Duží 2004], p. 29; italics by J.R.)[10]

First observe that *Duží does not really respect her "definition" or "preliminary characterization" because in them she talks about constructions of intensions* but *not about constructions of extensions* like the construction of existential quantifier ("the intension denoted by E"; [Duží 2004], p. 16). One may thus think that *now Duží suggests completely different theory of suppositions than she exposed before* – a theory which would like to explain *de dicto* / *de re* suppositions not only of (constructions of) empirical expressions but also of (constructions of) non-empirical expressions. Its possible quality I am discussing briefly below.

First let us realize that in constructions like (O3') the construction of existential quantifier determines the mapping which assigns truth-values to classes (but note that this mapping is not dependent on the state of world and time; it is an extension, not an intension). The character of this mapping is this: only one class, namely the empty class, obtains the truth-value false, other classes (which are nonempty) obtain the truth-value true. In constructions like (O3') the construction of an existential quantifier is the (first) part of a composition; the rest part of that composition serves to generate appropriate argument, i.e. a certain class. After obtaining the argument we obtain a value of that mapping ("existential quantifier") for that argument-class (however, independently on w, t). What is a contingent matter here is which class we obtain for the application of existential-quantifier-mapping. The propositional construction instructs to create a mapping from "all" values for *w*, *t*, and on different *w*-'s and *t*-'s we also obtain distinct classes. It is important to realize that in constructions like (O1') the construction of existential quantifier serves for the same purpose, i.e. for the determination of a proposition[11] – in the case of (O1') for the proposition to which is related an agent, because propositions are individuated by truth-values assigned to w, t couples; when we "forgot" to count with values of $\exists$ we obtain a proposition with distinct course of truth-values. *Duží's claim that "the truth-value of the proposition constructed by (O1') in w, t ... does not depend on the value of $^0\exists$" ([Duží 2004], p. 29) is thus simply false, in any context the existential quantifier does affect the truth-values of any construction which*

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[10] This comment appears also in [Duží 2004], p. 26, [Duží 2001a], p. 248.

[11] Moreover, when the construction $^0\exists$ is a subconstruction of construction like $^0[\lambda w \lambda t \, [^0\exists \, ...]]]$ it is such a part that participates on the specific identity of that construction (and then subsequently on the identification of it).

contain $^0\exists$. Hence, this Duží's "novel" theory of de dicto / de re suppositions is wrong – it is based on an unreal distinction.

Constructions of individuals and *de dicto / de re*

As for as constructions of individuals Duží suggested that attributing of being in *de dicto / de re* supposition is completely omitted:

> ^{0}W [individual-Wojtyla; J.R.] occurs (according to our definition) neither *de dicto* nor *de re*. This is not in accordance with Tichý's stipulation [Tichý 1988]. Tichý claims that proper names (or generally expressions denoting extensions) occur always *de dicto,* for they are not (and cannot be) subjected to intensional descent. Since we are digging in a way deeper and adding subtleties to Tichý's approach, we will keep our definition (and general characterization). ([Duží 2004], p. 22)

Again, in Duží's key definition 7 there is no word about constructions of other objects than intensions ("let C be an intensional construction", [Duží 2004], p. 15; see the respective quotation above). *One must then conclude that Duží has no real definition for the suppositions of constructions constructing extensions*.

One can then adopt Tichý's view that constructions of individuals are always in *de dicto* supposition (what is much "deeper and subtle" approach to the *de dicto / de re* than that of Duží). One can also generalize his claim about an example of a non-determiner "¬" for formulating *de dicto / de re* suppositions of nonempirical expressions, namely: every expression denoting an extension is always (and exclusively) in *de dicto* supposition – such an expression adverts to the construction-dictum it expresses, i.e. to the construction the expression is talking about, not to an unspecified (and unmentioned) res.[12] [13]
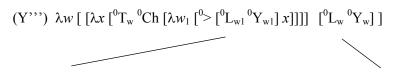
The marking of perspectives

Somebody can think that α-*renamings of variables* (which are unnecessary superfluous α-expansions) *would serve for special epistemological consideration – for marking which "perspective" has a speaker or an agent about occupancy of some office*. Duží explicitly suggested it in the following two quotations:

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[12] Compare [Tichý 1988], pp. 213-214.

[13] One can then easily answer which supposition is the "merged supposition" of constructions of individuals or numbers ("individuals or numbers are not functions, or perhaps only "quasi"-functions with zero arguments, hence *de dicto* and *de re* cases merge into one."; [Duží 2004], p. 22; the same in [2001a], p. 244).

the apparatus of Transparent Intensional logic (TIL), which enabled us to explicitly distinguish the "two worlds", reporter's perspective and believer's perspective, by means of explicit intensionalisation   ([Duží 2004], p. 32)

the variables *w, t* record the perspective – "world" of speaker, *w', t'* the perspective – "world" of whom an attitude is attributed.   ([Duží 2001a], p. 242)

Well, the distinguishing of an ascriber-reporter and an agent to whom some attitude is ascribed comes from Quine – "two ontologies, two worlds: that of the man in the attitude, however benighted, and that of our responsible ascriber of the attitude." ([Quine 1990], p. 69-70). Duží modified his suggestion by the relativization to propositional constructions. A typical example made by Duží is this:

(Y''') $\lambda w \ [\ [\lambda x \ [^0T_w \ ^0Ch \ [\lambda w_1 \ [^0> \ [^0L_{w1} \ ^0Y_{w1}] \ x]]]] \ \ [^0L_w \ ^0Y_w] \ ]$

$^0L, \ ^0Y$ (the former, Charles' perspective) - *de dicto*, $^0L, \ ^0Y$ (the latter, reporter's) - *de re*
                                                                          ([Duží 2004], p. 25)

This construction is (for her) the analysis of:[14]

(Y) Charles thinks that our yacht is longer than it is.   ([Duží 2004], p. 25)

First, I prefer disambiguated version of this sentence to which the analysis corresponds:

(Y') The length of our yacht is such that Charles believes that it is longer than it is.

and non-reduced construction of "the length of our yacht":

(Y'*) $\lambda w \ [\ [\lambda x \ [^0T_w \ ^0Ch \ [\lambda w_1 \ [^0> \ [\lambda w_1 \ [^0L_{w1} \ ^0Y_{w1}]] \ _{w1} \ x]]]] \ \ [\lambda w \ [^0L_w \ ^0Y_w]]_w \ ]$

One must ask whether her claim about the perspectives is clear enough. *What is a perspective? One may think (P1) that it is a relation of somebody to the thing which is a referent of certain (typical) description* (i.e. that somebody knows that this thing can be referred to by that description). Now the construction $[\lambda w \ [^0L_w \ ^0Y_w]]$ expressed by the first "the length of our yacht" (which is *de re* in the whole sentence) is in the speaker's perspective. Well, we immediately see, that the embedded sentence "it is longer than it is" is not in Charles perspective because he does not know – according to (P1) – to which truth-value this sentence (as a description of truth-values) refers. However, it is very natural to say that this sentence is in the perspective of the speaker (as well as it is for Charles). But is Charles really unaware which truth-value this sentence refer to? We do not know: he may or he may not know (the same holds for the speaker). Thus *from the whole sentence we cannot guarantee whether*

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[14] $T(hink) / (o \ \iota \ o_{\tau\omega})_{\tau\omega}$, $L(ength \ of) / (\tau\iota)_{\tau\omega}$, $> / (o\tau\tau)$, $(our)Y(acht) / \iota_{\tau\omega}$, $x \rightarrow \tau$.

speaker / agent knows to which truth-value (if any) the embedded sentence refers to. Generally, according to (P1) the whole sentence would be de dicto despite the fact that the speaker or the agent can know to which truth-value the whole sentence refers. (P1) must be wrong.

Similar consideration should arise if we will define *(P2): a perspective is a competence to decide (or to find) which thing is the referent of the description.* If some description is *de re* in the whole sentence the speaker may use it for a reference to a certain thing (res). If some other description is *de dicto* in the whole sentence the speaker does not use it such a way (i.e. for a reference). However, this is what we have already known before; and we still cannot derive that *de dicto / de re* suppositions say truly what is really known (or what is in the perspective) of an agent / a speaker.

Now I will show that (P1) is worse than (P2): the speaker pronouncing the sentence "Sukhi Turker is a man" hardly knows which class of individual is the class of the actual men, the actual referent of "man" – the class is "too large" and the speaker typically does not know it. Of course, the speaker can decide who is the man and who is not, (P2) is thus better. From all that observations we see that claims like:

> We have stated above that there is an important difference between ascribing belief attitudes in the *de re* way (from reporter's - ascriber's perspective) and in the *de dicto* way (fully in the competence of the person to whom the attitude is ascribed).   ([Duží 2004], p. 27)

cannot be correct: a speaker's sentence does not really imply "*de dicto* competence" of an agent. Moreover, *because the embedded as well as the whole sentence are not in the "competence of a speaker", according to (P2) a speaker is not competent to decide whether these sentences are true or false what is really not true. Thus intuitive understanding of perspectives in the sense of (P2) is also worthless.*

To sum up this subsection: *we tried to understand the notion of perspective which Duží did not explain enough. We have seen that the relating of the agent's / speaker's perspectives-competences to de re / de dicto suppositions of expressions would hardly be explicated sufficient way.*[15]

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[15] Of course, it is also dubious how "perspectives" can be formulated with respect to suppositions of non-empirical expressions – are numbers, functions, etc. in *de re* or *de dicto* perspective for an agent / a speaker?

THE EXCLUSIVE *DE RE* READING OF PASSIVE FORMS

Another claim related to the theory of *de dicto* / *de re* which Duží made in connection with *de dicto* / *de re* is the following one:

> the common belief at the equivalence of the active and passive form of a statement is not justified. The active form usually expresses the *de dicto* reading, whereas the passive form corresponds to the *de re* reading. ... [in] the case of propositional attitudes the passive form is exclusively *de re*. ([Duží 2003a], p. 251)

We will easily show that in the case of propositional attitudes (the same holds for notional attitudes) passive form[16] is not exclusively de re, hence Duží's claim does not hold.

Let us take for example the sentence

(D) Charles believes that the Pope is in danger.

with the analysis:

(D*$^{\text{dicto}}$) $\lambda w \lambda t$ [^{0}Believe$_{wt}$ ^{0}Ch [$\lambda w \lambda t$ [^{0}DA$_{wt}$ ^{0}P$_{wt}$]]]   // re: ^{0}Believe; dicto: other c's[17]

When the subject is not the agent of the verb, then the sentence is considered – by linguists – to be in the passive form. Now there are at least two main possibilities of passive forms. First consider that we focus an attention on the subject of the object of agent's attitude;[18] then we obtain a sentence ("*de re* reading" of (D)):

(D$^{\text{PS}}$) The Pope is such that Charles believes that he is in danger.

which is expressive of the construction:

(D$^{\text{PS*}}$) $\lambda w \lambda t$ [[λx [^{0}Believe$_{wt}$ ^{0}Ch [$\lambda w \lambda t$ [^{0}DA$_{wt}$ x]]] ^{0}P$_{wt}$]   // re: ^{0}Believe, ^{0}P; dicto: other c's

This construction is not β-reducible to (D*$^{\text{dicto}}$), β-reduction would be non-equivalent (see below Duží's reasons for that). (D) and (D$^{\text{PS}}$) are *not synonymous*. And we also see that *de dicto* and *de re* suppositions of expressions and constructions are different.

But the other passive form of (D) (this passive form accents the object of agent's attitude, viz. the proposition) is:

(D$^{\text{PO}}$) That the Pope is in danger is believed by Charles.

The analysis (variable p constructs o$_{\tau\omega}$-objects):

(D$^{\text{PO*}}$) $\lambda w \lambda t$ [[λp [^{0}Believe$_{wt}$ ^{0}Ch p]] [$\lambda w \lambda t$ [^{0}DA$_{wt}$ ^{0}P$_{wt}$]]]   // re: ^{0}Believe; dicto: other c's

---

[16] If we accept, despite the opinion of linguists, that these related forms should be called passive / active.

[17] These comments are made in accordance with Tichý's theory of *de dicto* / *de re*; it is about suppositions of constructions ("c's") with respect to the whole construction expressed by the whole sentence.

[18] This distinguishing of passive forms came from [Raclavský 2005], p. 262.

Raclavský, Jiří (2005): Duží's *De Dicto* and *De Re*: A Critical Review, *Pro-Fil* 6, 2. ISSN 1212-9097, http://profil.muni.cz/02_2005/raclavsky_duzis_dedictodere.pdf

Here *de dicto* and *de re* suppositions are preserved, (D^{PO}) is correctly β-reducible to $(D*^{dicto})$, (D) and (D^{PO}) *are equivalent*.

THE ANALYSIS OF A SUB-SENTENCE BY AN OPEN CONSTRUCTION AND THE CONTEXTUALISTIC ANALYZING

In [Duží 2000] (p. 377; and [Duží 2001a], p. 247) Duží has published a construction which seemed to her to be a result of β-reduction of α-expanded construction $(D*^{,dicto})$ and which became thus to be the analysis of *de dicto* reading of the sentence (D):

$$(D^*)\ \lambda w \lambda t\ [^0\text{Believe}_{wt}\ {}^0X\ [\lambda w \lambda t'\ [^0\text{DA}_{w't'}\ {}^0P_{wt}]]\ ]$$

After publishing [Duží 2001a] she realized ([Duží 2001b]) that this (D^*) is not equivalent with $(D*^{,dicto})$ because the construction expressed by the embedded sentence constructs another proposition than the corresponding construction which is a part of $(D*^{,dicto})$. We will meet her argumentation below, now I will put a very simple reason why (D^*) and $(D*^{,dicto})$ are non-equivalent.

Let us observe the construction expressed by the embedded sentence which is supposed to be the analysis of that embedded sentence. When we move it from the context of the construction (D^*):

$$(P^{*1})\ [\lambda w \lambda t'\ [^0\text{DA}_{w't'}\ {}^0P_{wt}]]$$

we can easily see that *it constructs different propositions dependently on the state of world and time*. Let us sketch a checking of this construction (w-'s, t-'s are values for the respective variables, let w_A, t_A be supposed as the actual world and the present time):

$$[\lambda w \lambda t'\ [^0\text{DA}_{w'\ t'}\ {}^0P_{w\ t}]]\ ]$$

w_1, t_1	w_A, t_A
: :	w_A, t_A
w_1, t_t	w_A, t_A
w_2, t_1	w_A, t_A
: :	w_A, t_A
w_2, t_t	w_A, t_A
w_n, t_1	w_A, t_A
: :	w_A, t_A
w_n, t_t	w_A, t_A

First consider that the actual Pope is identical with Joseph Ratzinger, i.e. that the value of the Pope office on that w_A, t_A is Ratzinger (the Pope office is defined for such w_A, t_A) − the

constructed proposition is true or false dependently on Ratzinger's being in danger.[19] But if we consider a world and time where nobody is the Pope, then "the Pope"-office is not defined on such w, t – the construction $^{0}P_{wt}$ is improper and the whole construction construct a proposition undefined for all w, t, i.e. a truth-valueless proposition. Well, one can easily find that *the analysis of*:

(P) The Pope is in danger.

must not to be this construction (P^{*1}) because this is an open construction – variables w, t are unbound in it. Thus there is no mystery in that (P^{*1}) constructs different propositions dependently on distinct values for *w* and *t*. Note also that there is no way how to correctly α-rename these variables, because the (closed) construction:

($P^{*2\alpha}$) $[\lambda w \lambda t' [^{0}DA_{w't'} \, ^{0}P_{w't'}]]$

is not α-equivalent to (P^{*1}). Note that *there is no reason why such open construction (P^{*1}) should be the analysis of a sentence which is fully disambiguated* (free of any indexical expressions). The correct analysis of sentence (P) is thus the closed construction:

(P^{*2}) $[\lambda w \lambda t [^{0}DA_{wt} \, ^{0}P_{wt}]]$

to which ($P^{*2\alpha}$) is α-equivalent.

Hence *Duží's suggestion how to analyze attitudes to propositions* we discuss *has another bad feature – analytical contextualism*. In "direct" ("normal") context as the analysis of (P) it is suggested (P^{*2}), but in "indirect" ("oblique") context of propositional attitude (P) it is suggested (P^{*1}). Due to the Tichý's model of language the language is a code (a mapping from coded gödelized expressions to meanings-constructions); but the contextualistic way of analyses makes such code being one-to-many mapping what is not the mapping a logician can accept. Thus one of the analyses (P^{*1})- (P^{*2}) must be repudiated – for the reasons given above, it should be the open construction (P^{*1}). Thus only (P^{*2}) is the analysis of (P) in direct as well as in indirect contexts.

What was Duží's reason for repudiating (D*) as an analysis of (D)? Duží rightfully observed that (D*) constructs a proposition which is "*a total* function, true or untrue in every world *w* and time *t*" ([Duží 2001b], p. 410). She argued approximately as follows: consider the state of world and time in which nobody is the Pope. As we have seen the embedded proposition would be undefined for (all) possible w, t, and if Xenia is related to such proposition, then the whole proposition is true; if she does not believe it, the whole

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[19] Similar considerations holds for other possible occupants of the Pope office; for example Karol Wojtyla.

proposition is false (the situation is similar for the cases where somebody is the Pope and is / is not in danger). Duží rightly claims that (D) denotes typically a partial proposition but (D*) does not construct such proposition; only from this Duží concluded that (D*) cannot be the analysis of (D) because truth-conditions for (D) are different. Above we have confirmed her opinion by the other reasons.[20]

THE PRINCIPLE OF EXISTENTIAL PRESUPPOSITION

Well, *Duží has also special reason why constructions like (D*) are wrong analyses of (D):*

> Since in the *de re* case there is the presupposition of the existence of 'the F', (DR) [i.e. our (D*'^{re})] is an accurate analysis, whereas (D*) does not respect this presupposition.
>
> ([Duží 2004], p. 21)
>
> P' [proposition constructed by (D*); J.R.], that is not a properly partial function any more; instead it is a *total* one, either true or false in any *w, t*. Hence the existential presupposition is *not* respected any more.　　　　　　([Duží 2004], p. 20)

But her reason is not intelligible with respect to (and thus justified by) her own formulation of the generally known principle about existential presupposition:

> The proposition that the "*de re* constituent" of a sentence exists is a presupposition of the sentence. In other words, the intension denoted by the expression occurring *de re* has to be instantiated in the given world/time (the reporter's perspective), otherwise the sentence (as well as its negation) does not have any truth-value at that world/time.　　([Duží 2004], p. 18)

(D*) evidently has such existential presupposition ("proposition"), namely (the construction expressed by) "There exists the Pope".

From that we see that *Duží's neglectful definition of existential presupposition confuses what one can call "the principle of existential consequence"*, i.e. (for example):

> (D*^{re}) or (D*) implies the construction expressed by the existential statement stating that there exists somebody who is the Pope (because ^{0}Pope is *de re* in (D*^{re}) and in (D*))

with "the principle of "existential presupposition", i.e. (for example):

> if the construction expressed by the existential statement claiming the existence of the Pope must construct true for (D*^{re}) being true or false (but not truth-valueless), then

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[20] Duží has made nearly a heroic attempt to repudiate (D) when she explained her one reason for many times ([Duží 2001a], p. 250, [Duží 2001b], p. 410, [Duží 2003a], p. 251, [Duží 2003b], pp. 71, 72, 73, [Duží 2004], pp. 18, 20, 21, 25, 30, 31, 31)

> this existential statement is an existential presupposition of (D*^{re}) (of course, (D*) has
> not this existential presupposition)

(realize that an existential statement is an existential presupposition "if something", not that existential statement is "in other words" the same as an existential presupposition, as Duží wrote).

For a documentation that Duží really confuses these two distinct principles let us cite:

> When a construction of an intension the F is used *de re*, there is an existential presupposition
> [here must be consequence; J.R.] on the existence of the F   ([Duží 2003b], p. 72)

The principle of existential presupposition is one of "*de re* principles" to which Duží refers the following way:

> In [Duží 2000] two important principles are formulated which hold in the *de re* cases, but not
> generally so in the *de dicto* cases. They are called the principles of:
> a) existential presupposition (commitment)
> b) intersubstitutivity of coreferential expressions   ([Duží 2004], p. 18)[21]

Well, as a matter of fact *one cannot find the principle of existential presupposition in her [Duží 2002]* because not the principle but only one (a bit confused) claim about one special sentence appears there.[22] *The similar holds for principle of intersubstitutivity of coreferential expression*:

> there are valid inferences when 'the murderer of Charles father' is replaced by another definite
> description with the same reference   ([Duží 2000], p. 374)[23]

Note also that her claim (as the name of the principle) suggests that in TIL substitution is syntactical operation with expressions what is not true (the substitution within TIL is an operation on constructions). The other drawback is that one cannot see what to do with non-empirical expressions (like 'Charles' etc.) – do they have a reference? Perhaps thanks to the relying on her alleged establishment of the principle of intersubstitutivity by coreferential expression in [Duží 2000] *Duží never suggested the principle of intersubstitutivity in her later articles at all*.

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[21] The same self-reference is made by Duží on p. 243 in [Duží 2001a].

[22] "the existential commitment, the existence of Charles' father, is a presupposition of the sentence [Charles hates the murderer of his father]" ([Duží 2000], p. 374).

[23] Similar particular comment is on p. 376 in her [Duží 2000].

THE INVALIDITY OF DE MORGAN LAWS

Although the previous parts of my critical review were "critical", "negative", this does not necessarily mean that this is the only aim of this paper.[24] Duží has discovered two interesting and important facts (well, not directly related to the *de dicto* / *de re*). I will be brief here; I will only report them and give a short comment. First:

> *De Morgan laws are not valid when using partial functions.*   ([Duží 2004], p. 31)

or:

> Many commonly accepted classical laws do not hold in case of logic involving partial functions   ([Duží 2003b], p. 56)

One must ask – is general logic based on so shaky grounds that such important laws do not hold? Duží shows several examples proving her claim; I will expose one of them:

> For instance the claim that there exists a pair of natural numbers such that their ratio is not a rational number is *false*, whereas the claim that it is not true that for all the pairs of natural numbers it holds that their ratio is a rational number is *true*. Formally, (Rat / ($o\tau$) - the class of rational numbers, Nat / ($o\tau$) - the class of natural numbers, variables *m, n* ranging over τ)
>
> (E) $[^0\exists\ \lambda\ m\ n\ ([^0\text{Nat}\ m] \wedge [^0\ \text{Nat}\ n] \wedge [\neg\ [^0\text{Rat}\ [^0{:}\ m\ n]]])]$ is not equivalent to
>
> (G) $[\ \neg\ [^0\forall\ \lambda\ m\ n\ (\ ([^0\text{Nat}\ m] \wedge [^0\text{Nat}\ n]) \supset [^0\text{Rat}\ [^0{:}\ m\ n]]\ )]\ ]$.
>
> We will show that *(E) constructs false, whereas (G) constructs true.*   ([Duží 2004], p. 31)[25]

What one has to ask here is whether (G) has really raised only by direct application of De Morgan Law on (E). The answer is "No": Duží has applied a) laws for transformations "$(...\wedge...\neg...)$" onto "$\neg\ (...\supset...)$"" and after that b) De Morgan Law.

Consider such valuation that *m* = 3 and *n* = 0; a) then the first conjunction returns true because 3 and 0 are natural numbers; b) 3 is not divisible by 0 thus the function ":" does not return a value (a particular number); c) about such "nihil" one cannot say it is or it is not a rational number, so "Rat" does not return a value (a certain truth-value) because no argument is given to that function; d) consequently the negation lacks the argument thus it will not produce a value; e) the main conjunction is missing a real argument, i.e. a couple (of

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[24] Well, I did not refer to Duží's correct claim ([Duží 2004, pp. 23-24]) that in the case of "knowing" one can correctly infer "*de re*" knowledge from the "*de dicto*" knowledge (and *vice versa*). However, this fact is stated already Jaakko Hintikka (see [Hintikka 1971], pp. 491-492; however, it seems that he knew that already before, perhaps in 1962 in the book "Knowledge and Belief").

[25] Informally it appeared in [Duží 2001b], p. 411.

its first member would be "true"), thus no value is yielded. Note, that when we have couples on which ":" is defined we obtain a rational number, the denial (that this number is not a rational number) is false and the main conjunction is also false. Generally, the class of such couples $<m,n>$ truly satisfying the condition is empty and "∃" returns false, thus (E) is false. This was correctly written by Duží and I will not document here that her comment on true-hood of (G) is also correct.

We have seen here the reason why our intuition – that there are couples of numbers (like <3, 0>) such both are natural numbers and the result of their division is not any (rational) number – was not fulfilled by the construction like (E). The reason was that the partial mapping ":" did not return a value on them and our predicate "being not a rational number" failed to produce the truth-value "false". Hence *we have to claim:*

> *In logic of partial functions we must not accept logical laws designed for the logic of total functions. Logical laws for the logic of partial functions must use "totalizing" predicate "True" which enables us to speak about functions sometimes not returning values.*

This is what has not been stated by Duží. A classical tautology formulated within transparent intensional logic is not, for example (*p* constructs propositions):

$$\lambda w \lambda t \, [^0\forall \, [\lambda p \, [\, [^0\neg \, p_{wt}] \, ^0\vee \, p_{wt}]]]$$

because such construction does not really return the truth-value "true" in every world and time – several propositions can be undefined for some w, t. The proper form of "the excluded middle" should be rather the following construction:[26]

$$\lambda w \lambda t \, [^0\forall \, [\lambda p \, [\, [^0\neg \, [^0\text{True}_{wt} \, p]] \, ^0\vee \, [^0\text{True}_{wt} \, p] \,]]]$$

the function "True" returns true when obtaining the truth-value true, however, it returns false in other cases. Now this is really a tautology.

If we turn back to (E), its "partial logic" version is:[27]

(E*) $[^0\exists \, \lambda \, m \, n \, (\, [^0\text{True} \, [[^0\text{Nat} \, m] \wedge [^0 \, \text{Nat} \, n]]] \wedge [\neg \, [^0\text{True} \, [^0\text{Rat} \, [^0 : \, m \, n]]]])$

which equivalent to:

(E*') $[^0\exists \, \lambda \, m \, n \, \neg \, (\, [^0\text{True} \, [[^0\text{Nat} \, m] \wedge [^0 \, \text{Nat} \, n]]] \supset \, [^0\text{True} \, [^0\text{Rat} \, [^0 : \, m \, n]]]])$

and this is equivalent to (applying De Morgan Law):

(G*) $\neg \, [^0\forall \, \lambda \, m \, n \, (\, [^0\text{True} \, [[^0\text{Nat} \, m] \wedge [^0 \, \text{Nat} \, n]]] \supset \, [^0\text{True} \, [^0\text{Rat} \, [^0 : \, m \, n]]]])$

---

[26] Somebody can think about tautologies for propositional constructions but it is not necessary for us to formulate such tautologies here (the relevant example: $\lambda w \lambda t \, [^0\forall \, [\lambda c \, [\, [^0\neg \, [^0\text{True}_{wt} \, ^2c]] \, ^0\vee \, [^0\text{True}_{wt} \, ^2c] \,]]]$).

[27] When working with mathematical propositions, one is better to explicitly handle with constructions of trivial intensions; I hope the reader will not be confused by my not using this practice (following in that Duží).

(E*) says what we are wishing to claim: there is at least one couple of numbers such that it is true that both numbers are natural numbers and it is not true the result of their division is a real number ($[^0\text{True}\ [^0\text{Rat}\ [^0:\ m\ n]]]]$ constructs "false" also for couples like <3,0>). Thus (E*) constructs truth-value "true" (for all w, t). Moreover, not all couples of numbers are such that if they are natural numbers, then it is true that their result of mutual dividing is a rational number, i.e. (G*) constructs true (for all w, t) as well.[28]

Despite of our correction, there is some good moral in Duží's example. However, it is more easily visible from her other example:

> Using for the sake of simplicity the simple concept ^0KF to construct the office KF of the King of France, a $\iota_{\tau\omega}$-object, K(ing) / $(o\iota)_{\tau\omega}$, our analysis is:
>
> (DD1') $\lambda w\lambda t\ ^0\exists w^*\ ^0\exists t^*\ [^0\neg\ [^0\text{K}_{w^*t^*}\ ^0\text{KF}_{w^*t^*}]]$ (^0K, ^0KF – *de dicto*)
>
> Constructs the impossible proposition, false in all *w,t* pairs (it is impossible that the King of France were not a king *"in the same world, time"*)   ([Duží 2004], p. 30) [29]

First, one may asks how an existing proposition can be impossible but suppose that Duží has chosen such term because this proposition obtains "false" for all arguments. Independently on that note that in worlds and times where there is no king of France, the proposition "the King of France is not a king" is without a truth-value (when there is no such individual being a king of France, then it cannot be a "king" or a "not king"), If the existential quantifier is understood as total mapping, it returns true. Thus the whole proposition constructed by (DD1') is constantly false. Now let us apply De Morgan Law:

> (DD2') $\lambda w\lambda t\ [^0\neg\ ^0\forall w^*\ ^0\forall t^*\ [[^0\text{K}_{w^*t^*}\ ^0\text{KF}_{w^*t^*}]]$ (^0K, ^0KF – *de dicto*)
>
> Constructs the necessary proposition, true in all *w,t* pairs (the composition $[^0\text{K}_{w^*t^*}\ ^0\text{KF}_{w^*t^*}]$, the King is a king, is *almost* true so to speak, i.e. true in all those *w*,t** pairs where $^0\text{KF}_{w^*t^*}$ is not *v*-improper, but in the other *w*,t** pairs it is *v*-improper, so that $^0\forall w^*\ ^0\forall t^*\ [[^0\text{K}_{w^*t^*}\ ^0\text{KF}_{w^*t^*}]]$ is false).   ([Duží 2004], p. 30)

We have seen that the existential quantifier returns true even if the argument is a "degenerated class"; however, the universal quantifier is not its "mirror" pendant – if at some w, t the constructed proposition is undefined, then the universal quantifier will not return "true", but "false". The denial posed by (DD2') is subsequently constantly "true", thus it is in fact a tautological proposition. Of course, such invalidity of De Morgan Laws must be – in logic dealing with partial functions – avoided by their suitable modification.

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[28] De Morgan Laws for the logic of partial function should be generally of form $\lambda w\lambda t\ [^0\exists\ [\lambda c\ [^0\neg\ [^0\text{True}_{wt}\ ^2c]]]]$ $^0\leftrightarrow\ [^0\neg\ [^0\forall\ [\lambda c\ [^0\text{True}_{wt}\ ^2c]]]]$.

[29] The whole this part was copied into [Duží 2003b], pp. 72-73.

To summarize this section: *Duží formulated a correct and interesting claim about the invalidity of De Morgan Laws when we work with partial functions*. Well, her ascertainment about the invalidity of classical logical laws in the logic of partial functions remained in some sense unfinished because she did not show which are the real logical laws for the logic of partial functions.

THE NON-EQUIVALENCY OF β-REDUCTION

In [Duží 2001b] Duží at first and then repeatedely[30] published a surprising claim about β-reduction – for her "general" β-reduction is not an equivalent transformation:

> Claim: *The accurate analysis of a de re attitude of the form X believes of the F that he/she/it has the property Prop is the non-reduced construction (DR), but the reduced (DRβ) is not equivalent to the former:*
>
> (DR) $\lambda w \lambda t$ [λx [$^0B_{wt}$ X [$\lambda w^*\lambda t^*$ [$^0Prop_{w^*t^*}$ x]]] O_{wt}] (O - *de re*)
>
> (DRβ) $\lambda w \lambda t$ [$^0B_{wt}$ X [$\lambda w^*\lambda t^*$ [$^0Prop_{w^*t^*}$ O_{wt}]]] (O - *de re*)   ([Duží 2004], p. 20)
>
> we enriched Materna's original definition by the "innocent" β_i-reduction, but warned against a "general" β-reduction. ... β-reduction is in a way too "strong" a transformation ... β-reduction is generally *not* an equivalent transformation when working with partial functions.
>
>   ([Duží 2004], p. 30)[31]
>
> *β-reduction is generally not an equivalent transformation when working with partial functions and must not be carelessly applied.*   ([Duží 2004], p. 31)
>
> *β-reduction is not an equivalent transformation in the case when we substitute for the variable ... composition ... which can be improper.*   ([Duží 2001b], p. 410)
>
> *Performing β-reduction on the construction having the form [λxλy [F x y] C], where x → α, y → β, F → (γαβ), C → α, is not an equivalent transformation in case of C being v-improper (even if F constructs a total function).*   ([Duží 2003b], p. 70)

The fact to which Duží refers is surely a shock for somebody who remember simply that in the typed λ-calculi the β-reduction is conceived as an equivalent transformation (β-conversion rule is usually understood as an axiom of λ-calculus, i.e. a tautology of an equivalence form.)

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[30] [Duží 2001a] p. 250, [Duží 2001b], p. 410, [Duží 2003a], p. 251,[Duží 2003b], pp. 71, 72, 73, [Duží 2004], pp. 1, 2, 4, 18, 20, 21, 30, 31.

[31] Duží repeatedly showed examples by which she documented such non-equivalence of "general" β-reductions; these are such as β-reduction of (D*^{re}) to (D*).

A remark. In the context of such considerations Duží suggested (firstly in [Duží 2001a], pp. 249-250, then [Duží 2004], p. 19) a definition of β-reduction which she called *"innocent" β_i-reduction*:

> *Definition 5 (β_i -equivalence of constructions*) Let C be a construction. By C($x_1/y_1,..., x_n/y_n$) we denote the result of collisionlessly [Tichý 1988, p.74, Def.17.2] replacing every occurrence of variable x_i in C by variable y_i (for $1 \leq i \leq n$). Then the construction $[[\lambda x_1...x_n \, C] \, y_1...y_n]$ is β_i - *equivalent* to the construction C($x_1/y_1,..., x_n/y_n$). (x_i, y_i, being of the same appropriate types). Let CC be a construction and let $[[\lambda x_1...x_n \, C] \, y_1...y_n]$ be a sub-construction of CC. Let CC' be like CC except that instead of $[[\lambda x_1...x_n \, C] \, y_1...y_n]$ it contains C($x_1/y_1,..., x_n/y_n$) as a result of β_i-reducing $[[\lambda x_1...x_n \, C] \, y_1...y_n]$. Then CC and CC' are β_i-*equivalent*. ([Duží 2004], p. 19)

In fact the first formulation of such definition was introduced by Alonzo Church as the I. rule of λ-conversion in [Church 1932]; I will cite his later formulation:

> To replace any part M_α of a formula by the result of substituting y_β for x_β throughout M_α, provided that x_β is not a free variable of M_α and y_α does not occur in M_α.[32]

([Church 1940], p. 60)

From this definition guaranteed by "the originator" of λ-calculi we also see that Duží's additional condition for providing β_i-reduction on subconstruction of some other construction ("Let CC be ...") can be easily omitted. Well, the third Church's rule of λ-conversion is not the target of Duží's attack, only the second rule concerned with the replacement of a variable by composition-application (usually restricted by a set of conditions) is suspected by her to be non-equivalent. (Note also that β_i-reduction is a very thin kind of β-reduction: it does not handle replacing of a hospitable variable by constructions like ^{0}X (trivializations), $\lambda w \lambda t$ [$..._{wt}..$] (closures), [...] (compositions), it handles only with replacing of variables by variables.)

Let us return to nonequivalent β-reduction and let us investigate how β-reduction is usually defined and whether such common definition avoids possible non-equivalency. Let us quote for example:

> The lambda calculus is the study of the set Λ modulo so called *β*-convertibility which is the least congruence relation $=_\beta$ axiomatized by[33]
>
> $(\lambda x.M)N =_\beta M[x:=N]$.

---

[32] Condition "and y_α does not occur in M_α" is usually formulated as "collisionless replacement". In Church's definition the "being of same types" is provided by subscripts marking types.

[33] M, N are some λ-terms, "M[x:=N]" signs "the replacing of x by N in M" (J.R.).

Here $M[x:=N]$ stands for the result of substituting N for the free variables of M.[34] In this notation the free variables of N are not allowed to become bound after substitution; for example $(\lambda y.x)[x:=yy] \neq^{35} (\lambda y.yy)$. By changing the names of bound variables one may obtain

$$(\lambda y.x)[x:=yy] \equiv (\lambda z.x)[x:=yy] \equiv (\lambda z.yy)$$

The notion of β-convertibility is an equivalence relation   ([Barendregt 1997], p. 3)

Read very carefully the last passage of this definition: "are not allowed to become bound after substitution". Let us investigate the following construction (or rather form of construction) C:

$$\lambda w \lambda t \, [\, [\lambda x \, [..._{wt}...[\lambda w \lambda t \, [..._{wt}...x]] \,]] \, {}^0D_{wt}]$$

The variable x is λ-bound in $[\lambda x \, [..._{wt}...x]]$ as well as in C, but free in x and in $[..._{wt}...x]$; but it is not important here. Rather we have to take care on the variables w, t which are free in themselves as well as in ${}^0D_{wt}$. C cannot be β-reduced to C':

$$\lambda w \lambda t \, [\, ..._{wt}...[\lambda w \lambda t \, [..._{wt}... \, {}^0D_{wt}]] \,]$$

because the variables w, t originally free in ${}^0D_{wt}$ would became λ-bound in $[\lambda w \lambda t \, [..._{wt}... {}^0D_{wt}]]$. Thus usual definition of a β-reduction seems to be correct. But what about the constructions whose subconstruction is "superfluously" α-renamed? Consider C^α (Duží's favorite form of constructions):

$$\lambda w \lambda t \, [\, [\lambda x \, [..._{wt}...[\lambda w' \lambda t' \, [..._{w't'}...x]] \,]] \, {}^0D_{wt}]$$

and its β-reduction is $C^{\alpha\beta}$:

$$\lambda w \lambda t \, [\, ..._{wt}...[\lambda w' \lambda t' \, [..._{w't'}... \, {}^0D_{wt}]] \,]$$

This is precisely what Duží showed firstly on p. 250 in [Duží 2001a] and then on p. 73 in [Duží 2003b], for example:

$$\lambda w \lambda t \, [\, [\lambda x \, [{}^0\exists \lambda w * {}^0\exists \lambda t * \, [{}^0\neg \, [{}^0K_{w*t*} \, {}^0KF_{wt}]]]] \, {}^0KF_{wt}] \qquad \text{(DR1)}$$

$$\lambda w \lambda t \, [{}^0\exists \lambda w * {}^0\exists \lambda t * \, [{}^0\neg \, [{}^0K_{w*t*} \, {}^0KF_{wt}]]] \qquad \text{(DR1}\beta\text{)}$$

In our example the variables w, t which are free in ${}^0D_{wt}$ are again free in $[\lambda w' \lambda t' \, [..._{w't'}... {}^0D_{wt}]]$. And above we have seen (the case of reducing to D*) that $C^{\alpha\beta}$ is not equivalent to C^α. However, one can think that due to the α-conversion, $[\lambda w' \lambda t' \, [..._{w't'}... {}^0D_{wt}]]$ can be α-reduced to $[\lambda w \lambda t \, [..._{wt}... x]]$ and w, t would become bound in the result; thus the β-reduction must not be executed; as a safe practice one may suggest to apply a β-reduction only to suitably α-converted constructions; perhaps it is better to state another formulation using terms like a "β-reduction is correctly executable if the reduction should not become non-equivalent due to superfluous α-renaming of some input constructions". Unfortunately, this demand is too

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[34] To be sure, consider $[{}^C\lambda x \, [{}^M...x...]]$; whereas x is free in M, it is λ-bound in C.

[35] Originally "$\equiv$" stands here but slashed.

strong. To see it, let us consider that ^{0}D constructs a total mapping. Then no problem would arise – C as well as C$^{\alpha\beta}$ constructs the same mapping. Briefly, it is a possible improperness of composition [^{0}D$_{wt}$] what is the real problem here – and that was exactly stated by Duží (as we quoted it above).

To sum up: the main "by product" of Duží's theory of *de dicto* / *de re* is *the fact that β-reduction can be non-equivalent transformation when we deal with partial functions. Well, as Solomon Feferman refers ([Feferman 1995]) this problem is known and studied within the partial lambda calculus.*

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[36] I have used his ftp://ftp.cs.ru.nl/pub/CompMath.Found/church.ps instead.

[37] All Tichý's articles are reprinted in Svoboda, Vladimír, Jespersen, Bjørn, Cheyene, Collin (eds.) (2004): *Pavel Tichý's Collected Papers in Logic and Philosophy*. Otago-Praha: Otago UP, Filosofia.