Are Concepts *a priori*?

Pavel Materna

Abstract

In [Laurence, Margolis 2003] the authors try - within their polemics against F.Jackson's views in [Jackson 1998] - to decide the question whether concepts are *a priori* (in their formulation "to be defined *a priori*"). Their discussion suffers - as a number of similar articles - from a typical drawback: some problem whose solution requires an exact notion of concept is handled as if the latter were quite clear. The consequence of this 'conceptual laxity' is that

a) the topic of the discussion is not very clear (what does the phrase 'concepts must be defined *a priori*' mean?);

b) the relevance of the Quinean criticism of the "second dogma of empiricism", i.e., of Quine's claim that "science sometimes overturns our most cherished beliefs" *and therefore* there is no sharp boundary between analytic and synthetic is uncritically accepted;

c) no distinction is made between the question whether the relation between an expression and its meaning is *a priori* and the question whether the relation between a concept and the object identified by the concept is *a priori*.

The present article intends to elucidate and then to answer the questions that can be asked when we say something like "concepts are *a priori* ".

0. Introduction

In [Laurence, Margolis 2003] the authors try - within their polemics against F.Jackson's views in [Jackson 1998] - to decide the question whether concepts are *a priori* (in their formulation "to be defined *a priori*". Their discussion suffers - as a number of similar articles - from a typical drawback: some problem whose solution requires an exact notion of concept is handled as if the latter were quite clear. The consequence of this 'conceptual laxity' is that

     a) the topic of the discussion is not very clear (what does the phrase 'concepts must be defined *a priori*' mean?);

     b) the relevance of the Quinean criticism of the "second dogma of empiricism", i.e., of Quine's claim that "science sometimes overturns our most cherished beliefs" *and therefore* there is no sharp boundary between analytic and synthetic is uncritically accepted;

     c) no distinction is made between the question whether the relation between an expression and its meaning is *a priori* and the question whether the relation between a concept and the object identified by the concept is *a priori*.

     The present article intends to elucidate and then to answer the questions that can be asked when we say something like "concepts are *a priori* ".

Concepts serve - among other things - to categorization of objects of our interest. Let us consider the case *whale* (only schematically, without considering the real process). Travelling

over the seas men frequently met animals distinguished by some interesting (even from the 'commercial' viewpoint) properties. Fulfilling the old Adam's task of naming (Genesis) the men named those animals, i.e., animals as the bearers of a cluster of the mentioned properties. Let the name be *whale* (Walfisch, kit, etc.). The name *whale* expresses what Frege would call 'sense' and denotes (not some definite class, as Frege would say, but) the property given by this sense. Let us just now accept the Churchian idea that 'sense' of an expression is a **concept** of the respective denotation. Thus we can say that the concept expressed by the name *whale* was a *way* to identifying the denoted property. We will see that a promising explication of this *way* consists in conceiving concepts as abstract procedures, a sort of *algorithm*,[1] that is. We can derive such an algorithm[2] in the following manner: Let *whale* be an abbreviation, i.e., a *definiendum*, where the *definiens* is a complex expression consisting of such subexpressions that are already used in the respective language. The definition means that the concept expressed by the definiendum is the concept expressed by the definiens; the latter is discovered as soon as an analysis of the definiens is realized. (This will be explained in details later.) As a schematic example the expression *the biggest fish in the sea* can be thought of as such a definiens. This expression, let it be named *E,* 'encodes' a procedure whose particular steps are given by the concepts expressed by the particular subexpressions of *E* and by the way they are combined together[3] (which again is determined by the grammar of the given language). Now in those olden days similar definitions did not respect zoological definitions of our time: either they did not respect zoological theories at all, or the zoological categories of that time differed from those ones we know nowadays. Thus let us suppose - for the purpose of our 'thought experiment' - that the expression *fish* was then to denote the property being an animal living in water, so that the respective concept can be derived by a logical analysis of the expression *being an animal living in water.* Let us ask: Which property has been identified by the procedure (concept) expressed by the expression

   the biggest animal living in the sea ?

It seems that the property is just that one denoted by the expression *whale,* as used then as well as nowadays.

Now zoologists come who classify animals according to much more fine-grained criteria than the people of the olden days, and they show that there is an important property that distinguishes one group of water animals from other groups of water animals. Yet what happens now? One would expect that *animal living in water*, i.e., *fish*, would denote what it denoted and that the newly respected property would get some new name. Language is, however, like the Holy Spirit: *flat ubi vult*. It is the 'new property', which gets the name *fish*. So the expression *fish* begins to express another concept. What happens now with the name *whale* ? Its definiens, i.e., *the biggest fish in the sea*, begins to express another concept and, as a result, to denote another property. The claim

C   *Whale is the biggest fish in the sea.,*

which was till then an analytic statement (since it reproduced the definition), seems now to be a false claim: Quine would say that it were impossible to define a boundary between analytic and synthetic expressions, since an empirical discovery might make the seemingly analytic

---

[1] See [Materna 1998], [Moschovakis 1994].

[2] We can imagine the most probable situation that the same name was connected with distinct concepts already in the olden days. One such possibility can be described as follows: the concept expressed by *whale* consisted in a simple procedure that immediately identifies the property. Such concepts are definable as ,simple concepts', see later. The other cases hold when *whale* is only an abbreviation - a ,definiednum' of some thinkable definition.

[3] See [Bolzano 1837], p. 244!

claim a synthetic one. For - Quine would say - we have firmly believed that **C** is true, being analytic, and now we see, being forced by experience, that it is false.

Quine's argumentation has been nearly dogmatically accepted because there has been a common belief that analyticity concerns *expressions*. It can be shown, however, that analytic/synthetic distinction concerns primarily the respective concepts and the objects identified by them. If so, then the expression **C** in the olden days expressed such a concept (of a proposition) that the proposition denoted *is* analytically true, and, of course, no experience can change this fact. Nowadays the expression **C** expresses another concept and denotes another proposition, which is no more true. To emphasize this distinction, let us provide **C** with indices: C_0 let denote the analytic proposition, C_n let denote the empirical false proposition. We can see that C_0 - unlike C_n - is immune to changes forced by experience: the Quinean claim that experience has influenced (and thus deleted) the analytic/synthetic distinction is simply false.

Many other examples of this kind can be adduced. Take, e.g., the word *planet*. The respective concept is a procedure that identifies a property. Once upon a time this property was defined in such a manner that the value of this property was a class of celestial bodies a member of which was, among other objects, the Sun. Consider now the sentence

Sun is a planet.

A Quinean approach[4] to determining the semantic status of this sentence would be: This sentence was once believed to be true, since it was a consequence of the definition of planets. Now the astronomers have shown that the sentence is false: Sun is not a planet. Thus again our experience has changed the semantic status of the sentence, which proves that the analytic/synthetic distinction is dubious.

Again: Not at all. When a concept of planets is given by the definition mentioned above, then no astronomer can refute the analytic truth of that sentence. What changed was not the truth-value: The expression *planet* began to express another concept, which no more identified the class containing the Sun. We began to use the word *planet* in a new sense, as expressing another concept and denoting another property (whose value in the actual world and time was another class, this time not containing the Sun). Concepts did not change, only the sentence has got another sense (meaning), i.e., it began to express another concept.

The absurdity of the claim that a new experience can refute a sentence whose denotation was an analytic proposition is easily demonstrable on our last example: Let *Sun* be the expression that denotes Sun. Let $planet_1$ denote that property whose value in the actual world is the set containing Sun and $planet_2$ denote the property defined by the modern astronomy. Consider the argument:

Sun is a planet$_1$

Sun is not a planet$_2$.

∴Sun is not a planet$_1$.

Obviously the conclusion does not follow from the premises; so the refutation of an analytic sentence (1st premise) by experience cannot be justified. By the way, the second premise is also analytic in the sense that it expresses an analytic concept and, therefore, denotes an analytic (true) proposition. The respective shifts of truth-values take place not because of some belief revision: the first premise remains to be (analytically) true; the concepts expressed by $planet_1$ and $planet_2$ simply determine distinct criteria (of 'planethood'). The reasons for 'inventing'[5] new concepts are to be looked for in the empirical area (history of

---

[4] I cannot know, of course, whether the ‚actual' Quine would agree, but the spirit of his philosophy - see, e.g., his [1953] - is obviously in harmmony with what follows.

[5] I would better say ‚discovering'; from the realist viewpoint this is a more precise formulation.

science). If, however, a sentence expresses such a concept of a proposition that the truth-value of the latter is (true or false but in any case) independent of the state of the world, i.e., can be unambiguously determined by the concept (i.e., a procedure!) itself then the Quinean theory of general influence of experience is abstruse.

In general, concepts cannot change, being abstract objects, only expressions can be connected with other procedures/concepts.

In [Laurence, Margolis 2003] we read that the discovery that space is non-Euclidean "breaks the apparent conceptual link between STRAIGHT LINE and THE SHORTEST DISTANCE BETWEEN TWO POINTS". Now we can ask: which concept is expressed by *straight line*? Is it the concept given by the respective Euclidean definition, is it the shortest distance between two points? Then no empirical claims like "Space is non-Euclidean" can change the analytic character of the sentence

The straight line is the shortest distance between two points.,

since it is stating the Euclidean definition. Well, what is the consequence of the discovery that space is non-Euclidean? Does this discovery force us to 'redefine' straight lines? The usual option is to make straight lines paths of a light ray. That such a new definition is counterproductive can be seen from the following quotation:

> [e]ither light rays travel along straight lines or they do not. ... If...they don't then it is obviously desirable to *state* this interesting fact. But this is precisely what we are unable to do once we have discarded the attribute of being a straight line in favour of the attribute of being a light-ray path.
>
> [Tichý 1988], p.277

So which 'conceptual link' has been broken due to the discovery of the non-Euclidean character of space? Do the expressions *straight line* and *the shortest distance between two points* express other concepts and denote other objects than as their denotations are defined in Euclidean geometry? If not, the above discovery does not break anything. If so, then the discovery is irrelevant as well: *straight line* and *the shortest distance between two points* denote then objects defined in a non-Euclidean geometry so that our sentence is false - but then what is primarily false is the respective proposition[6] and this proposition differs from the proposition in the Euclidean case.

In general, a *sentence can change its sense* (i.e., the concept of a truth-value - in the case of non-empirical sentences - or of a proposition otherwise); *if it was analytic possessing the original sense then no empirical discovery can overrule this fact.*

(Interestingly enough, we can frequently hear or read that our concepts develop. This may be in a sense true, viz. when concepts are considered to be mental entities (one of thoroughly elaborated theories of this kind can be found in [Bartsch 1998]), but as soon as we try to save the objective character of concepts (as Bolzano has done so many years ago) we immediately see the distinction between learning, acquiring, possessing concepts on the one hand and concepts themselves on the other hand; the former are mental processes so they can develop, of course, but the latter are abstract, 'ideal', immune to any process, any development. What does change/develop is language, and one form of its development consists in associating various distinct concepts with one and the same linguistic form.)

Now what has been said informally needs some more precise exposition. We will answer following questions:

What is concept?

---

[6] This formulation is not precise enough; in the case of pure geometry we can distinguish only truth-values and procedures („constructions", see later), no propositions in the sense of functions from worlds (and times) to truth-values are needed.

Concepts and expressions: what kind of connection?
Conceptual systems: their role in our categorization of objects
Definition: verbal vs. 'ontological' definitions

I.Concept[7]

Consider some 'universale' like the set of prime numbers. We will show that the common identification of concepts with 'universalia' (classically: Frege in his [1891], [1892]) is not in harmony with our using the term *concept*. If the set of prime numbers itself were a concept then our intuition, according to which *there may be more concepts of one and the same object*, could not be explained. (See, e.g., the introducing pages of [Bealer 1982].)

In our case we can identify the set of prime numbers for example by means of following two definitions:
1. *A prime number* is a (natural) number greater than 1 and divisible just by 1 and itself.
2. *A prime number* is a (natural) number divisible by exactly two numbers.

A most natural way how to describe this (and any analog) situation is to say that there are (at least) two distinct concepts that identify the set of prime numbers (in general: one and the same object). Assuming – as we should – that concepts differ from expressions of a language[8] we can say that the two concepts from our example are *meanings* ('senses', see Introduction of [Church 1956]) of the *definiens* of those definitions. At this moment we reduce our question to the question *what is the meaning of an expression*.

Undoubtedly many answers to this question can be found in the literature. The simplest one is offered by Quine: don't ask for meaning, ask for use. (For *meaning* is an 'obscure entity'.). Not very much of *semantic* theory can be built up after such an answer. So set it aside and look for other answers.

One of the oldest relevant answers (not using terms like *meaning, sense* etc.) is contained in the ingenious book [Bolzano 1837], where a consistent and very contemporary theory of concepts (*Begriffe*) is formulated. Bolzano argues that concepts are objective, independent of our mind, and they 'arise' as combining the members of a class called 'content' (of the concept in question). The idea of combining some objects (in this case 'simple concepts') to get the concept (p.244) can be very inspiring, as we will state later.

Frege's concept, as defined in [1891] and [1892], cannot be connected with sense (meaning), as known from his [1892a]; it is simply a (characteristic function of a) class, so that the counterintuitive consequence thereof (mentioned above) is that there could be no distinct but equivalent concepts. Therefore Church in [Church 1956] (Introduction) much more in the Fregean style placed concept where Frege had his 'sense'[9]: since sense should be a link between the expression and its denotation (*Art des Gegebenseins*) this Church's shift is closer to our intuition: concept is not what is *denoted* (as it was conceived of by Frege) but what is *expressed* and what leads to the denotation (so it is a 'way' to the denotation).

Another story can be read in Carnap's [1947], where Carnap rightly recognized that his intensions (as functions from state-descriptions) cannot play, in general, the role of the Fregean sense, because as mere functions, mappings, they are structurally neutral whereas at least some contexts are sensitive to the structure of the expression; therefore he defined 'structural isomorphism', so a kind of 'structured meaning'. His attempt can be criticized, and

---

[7] We explicate a *logically relevant* notion of concept: 'concepts' in the mentalistic sense are from this viewpoint mental objects, perhaps 'possessing concept' (see [Peacocke 1992].

[8] Why can we say that two expressions are synonymous: they share one and the same concept.

[9] Actually, he identified sense with concept: according to him the sense of an expression is a concept of the respective denotation.

it was, for example in [Church 1954], later by Tichý in [1988], but - interestingly enough - Carnap's argument that intensions do not satisfactorily play the role of sense has been frequently ignored: it is a commonly accepted wisdom that the best explication of Frege's sense is just intension in the sense of P(ossible-)W(orld)S(emantics). Notice that if we accepted this explication then we would have to admit that non-empirical (mathematical) expressions were devoid of any non-trivial sense. (For example, all true mathematical sentences would express one and the same 'sense', viz. the constant function that associates any possible world and time with the value **T**.)

The idea of structured meaning (sense) has been newly formulated by Cresswell in his [1975] and [1985]. Among the other proponents of structured meanings we should name Moschovakis (for example, his [1994]) or - from the intuitionist viewpoint - Fletcher in [1998]; the last two authors connected the structuredness with the notion of *algorithm*, which is the way that is closest to the conception that is defended by the present author.

In my opinion, the best apparatus for working out a general theory of concepts (which can play the role of meanings) can be found in Tichý's *transparent intensional logic* (TIL) as formulated in his [1988] and applied in most of his articles (see [2004]). I published therefore a study *Concepts and Objects* ([1998]) and a number of articles, where I try to show that a logically relevant explication of *concept* should be based on TIL. Some features of TIL are shared by Montague (see [Montague 1972]) but there are some important distinctions (referred to elsewhere), and I have chosen TIL.

In [Tichý 1988] and [Materna 1998] the main notions and principles of TIL are explained. We will therefore only informally reproduce the main points, referring for technical details (in particular, definitions) to the monographs mentioned above.

i) *Types.* Objects that can be talked about are classified in terms of ramified hierarchy of types. The *types of 1st order* are defined over the set of atomic types: truth-values (o), individuals (ι), time moments / real numbers (τ), possible worlds (ω)[10]. The other types of 1st order are sets of (partial) functions with domain $\langle \beta_1,...,\beta_m \rangle$ and range α for any types α, $\beta_1,...,\beta_m$, denoted by ($\alpha\beta_1...\beta_m$). *Types of higher orders* are definable after *constructions* are defined (see ii)). Then the set of constructions of order n ($*_n$) is a type of order $n + 1$, and sets of partial functions with arguments or values in $*_n$ for some n are types of order $n + 1$. So each thinkable object is a function: a nullary function for atomic types as well as for the types $*_1$, $*_2$, ... , an n-ary function for the other types. Classes an relations are also functions, viz. the respective characteristic functions. *Intensions* are functions from possible worlds to chronologies of a type so their type is (($\alpha\tau)\omega$) for any type α; abbreviated $\alpha_{\tau\omega}$. The type of the set of prime numbers is thus (oτ), propositions are of the type o$_{\tau\omega}$, properties of objects of type α are of the type (o$\alpha)_{\tau\omega}$, magnitudes (like the number of planets) are of the type $\tau_{\tau\omega}$, etc. *Non-trivial intensions* are functions whose values differ at least in two distinct worlds/times. The approach of TIL to objects denoted makes it clear that *empirical expressions denote non-trivial intensions*.

ii) *Constructions.* Whereas type-theoretical classification serves to categorization of objects denoted the notion of constructions explicates what should play the role of *meanings*. The idea of constructions can be described as follows:

Let us ask the following question: Consider a simple arithmetical expression, say,

7 - 5

The object denoted is obviously the number 2. But why? On the one hand we have an expression '7 - 5', on the other hand we have an extra-linguistic object, a number. How come

---

[10] The types o, ι, ω correspond to Montague's *t, e,* (non-type) *s,* respectively.

that the expression is linked just to the number? The rational core of the Fregean semantic scheme (see, e.g., [Frege 1892a]) consists in his attempt to answer this question. His 'sense' should have explained the character of this link. Yet Frege never *defined* this sense. TIL offers an exact definition, based on the following intuition: The expressions of a language *encode* some abstract procedures whose outcome is (in the better case) an object. Tichý formulates this viewpoint as follows:

> The notion of a code presupposes that prior to, and independently of, the code itself there is a range of items to be encoded in it. Hence…meanings cannot be conceived of as products of the language itself. They must be seen as logical rather than linguistic structures, amenable to investigation quite apart from their verbal embodiments in any particular language. To investigate logical constructions in this way is the task of logic. The linguist's brief is to investigate how logical constructions are encoded in various vernaculars.

[Tichý 2004, to appear]

Thus we can say (very roughly) that TIL is a theory of constructions[11]. The objective and algorithmically complex character of constructions is the core of this theory. Which particular constructions are chosen is not given by any dogma, and the choice should be dependent on the specific character of the given language. (This holds also of the choice of types, for example logical analysis of the language of arithmetic of natural numbers does not need the types ι, ω, and instead of τ we need the type ν (natural numbers).) The four important constructions in TIL (see [Tichý 1988], [Materna 1998]) are inspired by Church's ingenious introduction of λ-calculus in its typed version. The general idea is that it is mainly two operations which we need and use when we think: *defining functions* and *applying functions to arguments*. The former is usually called *abstraction*, the latter *application.* We can show it on our simple example.

There we have a function, viz. - (subtraction), and we apply it to the argument, i.e., the pair <7, 5>. Imagine a real procedure of computing. It is some sequence of steps that are realized and lead to identification of some object; in our case the number. This sequence of steps is a real time consuming process, whereas what we called *abstract* procedure is a timeless (and spaceless) abstract object, which we use when we realize the process. In our example the application is clearly given by the subtracting function. But how this function has been defined? Here we need not use the operation of abstraction (that makes up another component of the TIL system): we can identify the function in some 'immediate' way. This will be another component of the TIL system of 'basic constructions' (see 'trivialization'). This component is also needed when a construction 'is fed' by an object that is no construction (for example, the objects of the 1st order types are no constructions). Finally, for each member of the infinite hierarchy of types we have at our disposal (countably) infinitely many *variables* at our disposal. Here easily a misunderstanding can arise: We must not forget that the TIL constructions are objective, hence language independent (see the last quotation from Tichý). On the other hand, we are used from standard logical texts to define variables as letters, characters. The variables are considered in TIL to be a kind of (incomplete) constructions, which construct objects dependently on a total function called *valuation*. From this 'objectual' viewpoint (see [Tichý 1988]) the well-known letters like *x, y, z, …, p,q…,t, u,*

---

[11] Constructivists and intuitionists also use the term *construction*. The difference between constructions in TIL and the intuitionist constructions is that only the former are considered to be objective, i.e., independent of mind. They are not *created* but *discovered*. This philosophical distinction is accompanied by differing definitions in TIL and intuitionism. The idea of constructions as algorithmically complex entities is however common to TIL and intuitionism.

v,... are just *names of variables*: variables themselves remain to be extra-linguistic constructions.

We have very roughly characterized the four constructions we will use in our text. Now we introduce notations and try to suggest the ideas of particular definitions, which are to be found in the literature mentioned above.

a) *Variables* They *v*-construct objects, where *v* is a parameter of valuations.

b) *Trivialization,* ^0X, where X is an arbitrary object (of any type). It constructs X without any change.

c) *Composition,* $[XX_1...X_m]$, where X, $X_1,...,X_m$ are constructions. It constructs (in general, *v*-constructs) the value of the function (*v*-)constructed by X on arguments (in general *v*-)constructed by $X_1,...,X_m$. Functions are partial, so in some cases the composition *v*-constructs nothing, is *v-improper*.

d) *Closure,* $[\lambda x_1...x_m\ X]$, where $x_1,...,x_m$ are distinct variables and X is a construction. It *v*-constructs a function from the tuples of objects the variables range over to the objects *v'*-constructed by X when the occurrences of *x*s are replaced by the given tuple of objects. (Roughly.)

Thus when a composition (which corresponds to applying a function to arguments) applies the function of a type $(\alpha\beta_1...\beta_m)$ to arguments of the types $\beta_1,...,\beta_m$ then the type of the constructed object will be α. And when the variables in the closure range, respectively, over the types $\beta_1,...,\beta_m$ while X *v*-constructs objects of the type α, the *v*-constructed object will be of the type $(\alpha\beta_1...\beta_m)$.

An important reminder: constructions are not what is constructed, they contain their components together with the roles of them. They are no 'amalgams' of the particular 'steps' as the objects constructed are.

Now we can show examples of some simple constructions that can be associated with some expressions as their meanings (the analysis will be necessarily a little simplified).

Consider the sentence

The Pope is ill.

First of all we have to determine types of particular (sub)expressions. This type-theoretical analysis precedes the analysis proper. Thus we have

The Pope / $\iota_{\tau\omega}$; the Pope is not the same as the (actual) bearer of the papal office. It is just an 'individual office' (Tichý's term; we can also speak about 'individual roles').
If, for example, somebody wants to become the Pope, then it does not mean that he wants to become Wojtyla. Thus the type will be an intensional type, i.e., a function from possible worlds to a chronology of individuals. Take a world and time and you will get at most one bearer of the office Pope.

(*being*) *ill* / $(o\iota)_{\tau\omega}$; being ill is a property of individuals; the intension in question associates every world and time with a (maybe empty) class of individuals.

The Pope is ill./ $o_{\tau\omega}$; the sentence is an empirical sentence, so it denotes (not a truth-value but) a proposition.

Thus our task is to combine the constructions of objects of the types $\iota_{\tau\omega}$ and $(o\iota)_{\tau\omega}$ to get a construction of an object of the type $o_{\tau\omega}$ and to ensure that the latter will be such a proposition that our sentence actually denotes it (in other words, the resulting construction should construct such truth conditions which every normal speaker will connect with our sentence.

The object constructed should be a proposition, i.e., an $o_{\tau\omega}$-object. So we need to construct a *function* from ω to a chronology of truth-values, i.e., to a function of the type $(o\tau)$. We use the closure and write

1) $\lambda w\, \lambda t\, A$,

where w, t are variables ranging over ω, τ, respectively, and A is a construction that v-constructs truth-values and contains only w, t as free variables. How do we get this construction? *A* must obviously contain the constructions of the Pope and of being ill. The latter is predicated about an individual but the Pope is an individual role. An individual role cannot be ill, but the sentence says that its bearer is. Thus we have to apply the property first to a possible world and then (the result) to a time moment: we will get an individual that is the Pope in the given world at the given time moment. Which of the worlds is the actual one and which of the time moments is the present one cannot be given by a logical analysis, so we use the respective variables $w, t,$ which will be abstracted over in 1). We get

2) $\lambda w\, \lambda t\, [\, [[^0\text{Ill}\, w]t]\, [[^0\text{Pope}\, w]t]\,]$,

and using an obvious abbreviation we can write

2') $\lambda w\, \lambda t\, [^0\text{Ill}_{wt}\, {}^0\text{Pope}_{wt}\,]$,

which constructs just the truth conditions that make our sentence true: it will be true in such worlds w and time moments t where the individual who is in w at t the Pope is a member of the class that is the value of the property being ill in w at t. (Thus, for example, in such a w and t where there is no Pope the constructed proposition is undefined, i.e., neither true nor false.)

We must be always aware of the fact that constructions are not the artificial means of their fixation but abstract *procedures*, say, algorithms that had to be fixed by this artificial notation. These algorithms work with (variables and) the simplest steps, which are trivializations of some objects-non-constructions (here: ^0Ill, $^0\text{Pope}$). Let us try to call such trivializations *simple concepts*. Then we could say that the concept $\lambda w\, \lambda t\, [^0\text{Ill}_{wt}\, {}^0\text{Pope}_{wt}]$[12] 'arises' by means of combining - in a well-defined way - some simple concepts. The *set* of the simple concepts (here $\{^0\text{Ill}, {}^0\text{Pope}\}$), as well as the *pair* of them ($<^0\text{Ill}, {}^0\text{Pope}>$[13]) differ essentially from the above construction, which is a definite way how to handle both simple concepts.

To propose a viable definition of concepts we need the notion of bound variables. Only briefly (otherwise see [Materna 1998]):

Let C be a construction that contains variables $\xi_1,..., \xi_k$.. All of them are *bound by trivialization* (^0bound) in the construction ^0C.

Let C be $[\lambda\xi_1... \xi_k.\ X]$. Then the variables $\xi_1,..., \xi_k$ are *λ-bound* in C unless they are ^{0}bound in X.

Example: In $\lambda x\, [^0+\, x\, {}^01]$ (x ranges over natural numbers)[14] the variable x is λ-bound.
        In $^0[\lambda x\, [^0+\, x\, {}^01]]$ the variable x is ^{0}bound.[15]

Let a variable be neither ^{0}bound nor λ-bound in a construction C. Then we say that it is *free* in C.

A construction is called *closed* iff no free variables occur in it.

---

[12] We accept Church's generalization: according to him also *sentences* express concepts. The concept expressed by a sentence identifies a truth-value (for Frege always, for TIL only if the sentence is not empirical) or a proposition (in the empirical case).

[13] Where TIL has constructions, Cresswell has just tuples.

[14] Notice that in this case the construction constructs the *successor*.

[15] This construction constructs the construction $\lambda x\, [^0+\, x\, {}^01]$.

The key definition:

A *concept* is a closed construction.

Remark I. The motivation for this definition is the following one: Concepts, as the ways to the object, have to be procedures. We can interpret the fact that a concept identifies an object as the fact that the respective construction constructs the object. On the other hand, a concept should identify an object without 'waiting for a parameter'. Thus we would say that the expressions

a father, the father of , the father of A.Einstein

express concepts (the respective constructions do not contain any free variable). The expression *the father of x* cannot identify a definite object before some valuation associates *x* with an individual. (Notice the distinction between *the father of x* and *the father of* : the construction underlying the former expression contains the free variable *x*, whereas the construction expressed by the latter binds all variables - it begins with $\lambda w\ \lambda t\ [\lambda xy\ ...\ .)$

Remark II. The above definition of concepts has to be made more precise. The problem is that distinct constructions of a certain kind represent one and the same concept. A simple example: a concept of positive numbers is represented by infinitely many constructions like

$\lambda x_1\ [^0> x_1\ ^0 0]$, $\lambda x_2\ [^0> x_2\ ^0 0]$, *... ad infinitum.*

A good solution of this problem can be found in

[Horák 2001]   Horák, Aleš: The Normal Translation Algorithm in Transparent Intensional Logic for Czech. PhD Thesis, Brno.

This solution (based on a correct 'normalization') makes it possible to use our definition without important problems.

II. Concepts and expressions

As soon as we define concepts as being abstract procedures (closed constructions) we can see many classical problems from a distinct angle that it is usual. (Some such new views have been suggested in *Introduction*.)

First of all, concepts-procedures are no mental objects (being *abstract*). This is a Bolzanian tradition, and when the contemporary *logicians* speak about concepts they share this view. Distinguishing between concepts and, e.g., possessing concepts is highly important (see also [Peacocke 1992]). This claim is the first opportunity to formulate our attitude to the question *Are concepts a priori?*

This formulation of the question is not very clear. The authors of [Laurence, Margolis 2003] think (p.254) that the positive answer would be "concepts must be definable *a priori*" and that "it's philosophy's job to furnish the definitions". According to them Quine and Putnam "convinced many philosophers that this is a mistaken view", and they reproduce later some arguments supporting this Quinean criticism.

But first of all, what does it mean that concepts must be *definable...* ?

To answer this question we have to insert two subchapters into the present chapter.

IIa. Conceptual systems

First of all, let us define *simple concepts.*

Let X be an object that is no construction. Then ^{0}X is a *simple concept. -*

Let $C_1,...,C_k$ be simple concepts. The set

$$\{\ C_1,...,C_k\ \}$$

will be called the set of *primitive concepts* of a *conceptual system*. The latter unambiguously generates (using variables) the set

$$\{C_{k+1},\ ...\ \}$$

whose members are concepts (distinct from the primitive ones) the simple subconcepts of which are only the members of $\{\ C_1,...,C_k\ \}$[16]. For the sake of simplicity we will call *conceptual system* any set

$$\{\ C_1,...,C_k\ \} \cup \{C_{k+1}, ... \}$$

as defined above. The members of the set $\{C_{k+1}, ... \}$ can be called *derived concepts*.

As a most simple example consider the set that contains the concepts of all truth functions

$$\{^0\neg,\ ^0\wedge\} \cup \{\lambda pq\ [^0\neg\ [^0\wedge\ [^0\neg p][^0\neg q]]],\ ...\ \textit{ad infinitum} \}$$

The notion of conceptual system is highly abstract. For example, we cannot suppose that a given natural language at a given stage of its development were based on some unique conceptual system. All the same, the notion enables us to handle and elucidate some well-known problems connected with semantics of natural languages. One such complex of problems is connected with Kuhn's relativistic tendency (for a critical analysis of Kuhnian ideas see [Sankey 1997]). Already now we can state that the incommensurability thesis can get a rational interpretation in terms of dependence of formulating scientific claims on the choice of a conceptual system. But we have to strictly distinguish between choosing a concept(ual system) and choosing (a system of) expressions. Ignoring this difference can lead to an embarrassing doubletalk.

Now we can return to the question of 'definability of concepts'.

IIb. Verbal vs. 'ontological' definitions.

We are used to say that we define *an expression.* Indeed, the classical *verbal definition* is an expression of the form

$$\textit{Definiendum} =_{df} \textit{Definiens,}$$

where *Definiendum* is a simple expression and *Definiens* is a complex expression containing only 'already known' expressions. We then say that what is defined is *Definiendum*, i.e., an expression: *Definiens* determines the meaning of this 'left side' expression.

Actually, the semantics of verbal definitions is much deeper. The expression 'already known' above suggests a kind of relativity: at first sight it seems that this point can be whisked away by referring to the way in which it is solved in the axiomatic systems, viz. by introducing 'primitive terms', which then unambiguously determine the class of 'already known' expressions. But setting aside the fact that even the primitive terms of an axiomatic systems should express some concepts (at least if we are not content with Hilbert's answers to Frege) we must state that referring to primitive terms is not usable as soon as we come to natural languages.

So let us first come over to the world of (extra-linguistic) concepts. Let **S** be a conceptual system with primitive concepts $C_1,...,C_k.$. Let us omit from the set of the derived concepts of **S** those concepts that are result of cumulating trivializations[17]. Thereafter we can call the set of those objects which are constructed by **S** *the area of S.* Now we can say that the derived (i.e., complex) members of **S** *define* the members of the area of **S**[18]. Every derived concept of a conceptual system is called an *ontological definition* unless it is strictly empty, i.e., unless it (the construction!) is improper.

Returning to the verbal definitions we can say that explaining their semantics has to be done in terms of conceptual systems. In the case of classical equational definitions (see

---

[16] What is a subconstruction (and thus also what is a subconcept) of a construction is easily definable. See, e.g., [Materna 1998].

[17] Let C be a concept. Theoretically, we can ‚create' infinitely many trivializations: ^0C, ^{00}C, ... We omit just such infinite sequences from the set of derived concepts.

[18] The *definiens* of a verbal definition, as well as an ontological definition must be complex. Therefore only *derived* concepts of the given conceptual system can be called (ontological) definitions.

above), the *definiens* expresses some derived member D_S of a conceptual system S, so that it *defines the object* that is *ontologically defined* by D_S. The *Definiendum* gets its meaning from the *Definiens*, so that it denotes just what is denoted by the *Definiens*, but not only that: the latter endows the *Definiendum not only with its denotation, but also with its meaning,* i.e., the concept expressed by the *Definiendum* does not differ from the concept expressed by the *Definiens. Thus the 'left side expression' is a simple expression but it expresses a complex concept.* (see [Materna 1999]).

Verbal definitions serve essentially to introducing *abbreviations*. If we use such an abbreviation and somebody does not understand we have to offer a (verbal) definition. If our partner does not understand again then it means that (s)he does not understand some term(s) in the *Definiens*. The process can continue and we get Happy End as soon as every member of the augmented *Definiens* has been understood. The right conceptual system has been found.

A consequence of our conception is that if we wished to write down a construction that would be an analysis of a definition we would always get a concept of a trivial proposition **TRUE**. A simple example:

In a conceptual system with primitive concepts $^0\neg$, $^0\wedge$, ^{0}Man, ^{0}Adult, ^{0}Married, where the types of the objects are

$\neg$ / (oo), $\wedge$ / (ooo), Man, Adult, Married / $(o\iota)_{\tau\omega,,}$

we have a following derived concept (*x* ranges over ι):

$\lambda w\,\lambda t\,\lambda x\,[^0\wedge\,[^0\text{Man}_{wt}\,x][^0\wedge\,[[^0\text{Adult}_{wt}\,x][^0\neg\,[^0\text{Married}_{wt}\,x]]]]]$.

This concept is expressed by the English expression

a man who is adult and not married;

now an abbreviation has been introduced in English by the definition

a bachelor is (or: $=_{df}$) a *man who is adult and not married*;

a TIL style logical analysis of this definition has to respect the character of definitions: namely that they lay down the meaning of *definiendum* by identifying it with the meaning of *definiens* (= / (o $(o\iota)_{\tau\omega}$ $(o\iota)_{\tau\omega}$)):

$[^0= [\lambda w\,\lambda t\,\lambda x\,[^0\wedge\,[^0\text{Man}_{wt}\,x][^0\wedge\,[[^0\text{Adult}_{wt}\,x][^0\neg\,[^0\text{Married}_{wt}\,x]]]]]]$

$[\lambda w\,\lambda t\,\lambda x\,[^0\wedge\,[^0\text{Man}_{wt}x]\,[^0\wedge\,[[^0\text{Adult}_{wt}\,x][^0\neg\,[^0\text{Married}_{wt}\,x]]]]]]]$.

In general: any verbal equational definition expresses a concept that identifies the truth-value **T.** Not only that: Any claim that states that the object (intension!) has some of its requisites[19] is also analytic (and, of course, *a priori*); the requisites are derivable from the definition.

Example: Consider the sentence

Every bachelor is a man.

Having $\forall$ / (o(oι)) and $\supset$ / (ooo) we can offer the following analysis:

$\lambda w\,\lambda t\,[^0\forall\,\lambda x\,[^0\supset\,[^0\wedge\,[^0\text{Man}_{wt}\,x][^0\wedge\,[^0\text{Adult}_{wt}\,x][^0\neg\,[^0\text{Married}_{wt}\,x]]]]\,[^0\text{Man}_{wt}x]]]$;

this concept constructs a trivial proposition **TRUE** (whose value is **T** in all worlds and times). What if we instead accepted the following analysis?

$\lambda w\,\lambda t\,[^0\forall\,\lambda x\,[^0\supset\,[^0\text{Bachelor}_{wt}x][\,^0\text{Man}_{wt}x]]]$?

In other words, what happens if we believed that the simple expression *bachelor* expresses a simple concept? Being simple the given concept identifies the respective object (in our case the property bachelor(hood)) without calling any other concept (of being adult etc.). Thus

---

[19] A property Q is a requisite of a property P iff any bearer of the property P has also the property Q in all worlds and times. By analogy for the case where P is another intension than property. See [Tichý 1979], in [Tichý 2004].

we - knowing the respective language - will get the result that under any conditions our sentence is true but the simple concept ^{0}Bachelor 'does not make explicit' the reason thereof.

A thought experiment: imagine that during some development of English the word *bachelor* will be used more generally: any adult - man or woman - will be called so if (s)he is not married. The sentence

Every bachelor is a man.

is then a (probably) false empirical claim. The Quineans would say: Experience has taught us that the seemingly analytic truth can be falsified. We should know by now that such an interpretation would be false. The 'new' concept (better: the concept newly associated with the word *bachelor*) makes from our sentence a homonym. The analytic *a priori* character of the original sentence is not in the least influenced by the linguistic change.

Now we could say: If a concept C constructs a proposition in such a way that a requisite of an intension *P* is predicated about *P* then C is an *a priori* concept. (Notice that such cases fall under the Kantian definition of analyticity.) Such a result would be very poor, of course. A generalization is thinkable: If a concept C constructs a mathematical object then we would surely like to say that C is *a priori.* But what about empirical concepts? Is a concept connected with the expression *planet* or *whale* also *a priori?* It is just empirical concepts whose *a priori* character has been questioned.

If concepts are construed as being procedures (e.g., constructions in the sense of TIL) and if they can serve as meanings of expressions then answering our question requires understanding that there are two questions here:

1. Is the relation between an expression and its meaning (concept!) *a priori*?

2. Is the relation between a concept and the object identified by it *a priori*?

Let us first answer *the question* **1.**

A conceptual analysis in our sense is a matter of semantics. We suppose that the linguistic convention determines meanings (and thus the denotations) of the given expressions (at the given stage of development of the language in question) and that from the viewpoint of semantics this is simply given (semantics - unlike various theories of language - is not an empirical discipline).[20] Thus a logical analysis of an expression E tries to find the concept encoded by E by the supposed convention (see [Duží, Materna 2003]) and is based exclusively on the respective code, without referring to empirical circumstances. Thus the relation between expressions and their meanings (concepts) is *a priori.*

Now *the question* **2.**

A simple empirical concept ^{0}X constructs - according to our definitions - exactly the object X. No experience can change the fact that X is an intension, and intension as a function whose domain are possible worlds is always *a priori* (unlike the *values* of the intension).

Now what is the kind of compound constructions that are possible meanings of empirical expressions: Since they construct intensions they are given by the scheme

$\lambda w \lambda t$ X,

where X is a construction containing *w, t* as the only free variables. Again, no compound empirical concept can construct an entity of other kind than intension. Thus the relation between concepts and the objects they construct is also *a priori.*

---

[20] „We assume, of course, a normal linguistic situation, in which communication proceeds between two people who both understand the language. Logical semantics does not deal with other linguistic situations." [Tichý 1966], in [Tichý 2004].

References

[Bartsch 1998]   R.Bartsch: *Dynamic Conceptual Semantics.* CSLI Publications & FoLLI

[Bealer 1982]   G.Bealer: *Quality and Concepts.* Clarendon Press, Oxford

[Bolzano 1837]   B.Bolzano: *Wissenschaftslehre.* Sulzbach

[Carnap 1947]   R.Carnap: *Meaning and Necessity.* Chicago UP

[Church 1954]   A.Church: Intensional Isomorphism and Identity of Belief. *Philosophical Studies,* Vol.5, 65-73

[Church 1956]   A.Church: *Introduction to Mathematical Logic I.* Princeton

[Cresswell 1975]   Max J.Cresswell: Hyperintensional Logic. *Studia Logica XXXIV,1,*25-38

[Cresswell 1985]   Max J.Cresswell: *Structured Meanings.* MIT Press, Cambridge, Mass.

[Duží, Materna 2003]   M. Duží, P.Materna: Parmenides Principle (An Analysis of Aboutness. In: Childers, T. and Majer, O., eds.: *The Logica Yearbook* 2002, 159-178

[Fletcher 1998]   P.Fletcher: *Truth, Proof and Infinity.* Kluer AP, Dordrecht/Boston/London

[Frege 1891]   G.Frege: *Funktion und Begriff.* H.Pohle, Jena

[Frege 1892]   G.Frege: Über Begriff und Gegenstand. *Vierteljahrschrift für wissenschaftliche Philosophie 16,* 192-205

[Frege 1892a]   G.Frege: Über Sinn und Bedeutung. *Zeitschrift für Philosophie und philosophische Kritik 100,* 25-50

[Jackson 1998]   F.Jackson: *From Ethics to Metaphysics.* Oxford UP

[Laurenc, Margolis 2003]   St.Laurence, E.Margolis: Concepts and Connceptual Analysis *Philosophy and Phenomenological Research, LXVII, 2,* 253-282

[Materna 1998]   P.Materna: *Concepts and Objects.* Acta Philosophica Fennica 63, Helsinki

[Montague 1972]   R.H.Thomason, ed.: R.Montague: *Formal Philosophy.* Yale UP, New Haven

[Moschovakis 1994]   Y.N.Moschovakis: *Sense and denotation asalgorithm and value,* In: J.Väänen and J.Oikkonen, eds.: *Logic colloquium'90* vol.2 Association for Symbolic Logic, Lecture notes in Logic, 210-249

[Peacocke 1992]   Chr.Peacocke: *A Study of Concepts.* The MIT Press

[Quine 1953]   W.v.O.Quine: Two Dogmas of Empiricism. In: *From a Logical Point of View.* Harvard UP, Cambridge, Mass.

[Sankey 1997]   H.Sankey: *Rationality, Relativism and Incommensurability.* Ashgate

[Tichý 1966]   P.Tichý: K explikaci pojmu ´obsah věty´. *Filosofický časopis 14,* 364-372 (On explicating the notion of the content of a sentence. See [Tichý 2004]

[Tichý 1979]   P.Tichý: Existence and God. *The Journal of Philosophy 76*, 403-420. See [Tichý 2004]

[Tichý 1988]   P. Tichý: *The Foundations of Frege's Logic.* deGruyter

[Tichý 2004]   P.Tichý: *Collected Papers in Logic and Philosophy.* Dunedin-Prague, to appear