

STUDY

FACILITATION STRATEGIES FOR DEVELOPING MATHEMATICS TEACHERS' NOTICING OF COMMUNICATIVE ORCHESTRATION

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ABSTRACT

Mathematics teachers develop in tandem via the professional competence known as noticing. In this study, "noticing" is defined as mathematics teachers' identification and interpretation of communicative orchestration. Researchers have not yet fully investigated how the actions performed by a facilitator can influence noticing of communicative orchestration; therefore, the objective of this study is to characterize the facilitation strategies used by the facilitator during a professional development program (PDP) designed to support mathematics teachers' noticing of communicative orchestration. Building on a prior study that indicated that teacher noticing of communicative orchestration improves when discussing it within a group, we analyzed the facilitation strategies of a group discussion facilitator to determine how these actors could help five teachers to improve their noticing competence. We selected and adapted facilitation strategies (van Es, 2009) through actions that we analyzed in sessions in which a facilitator guided whole group discussions. Our results revealed the predominance of the "prompter" facilitation strategy, related to the identification of communicative orchestration, and the "proposer" facilitation strategy, related to its interpretation. The "builder" and "critic" facilitation strategies, which involve using evidence and challenging participant interpretative positions about communicative orchestration, were less frequent. These results are examined considering the levels of identification and interpretation attained by the teachers. Finally, the limitations of the study are described.

KEYWORDS

teacher noticing; facilitator; facilitation strategies; communicative orchestration; professional development

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Introduction

The role of professional development program (henceforth PDP) facilitators is fundamental: by helping teachers to improve their practices, student learning is ultimately enriched (Borko, 2004). PDP facilitators play one or all of the following roles: designing the PDP, inviting teachers to participate, implementing the PDP, and evaluating the PDP and/or conducting research based on it (Lange & Meaney, 2013; Nieman et al., 2023; Ricart et al., 2022). Each PDP, regardless of its characteristics, is led by at least one facilitator who generates discussion opportunities for the teachers who participate and collaborate in the learning process (Segal et al., 2018). There are multiple types of facilitators, including those who serve as mentors for schoolteachers, known as teacher leaders (Amador & Weiland, 2015; Amador, 2020). Other facilitators support teachers in activities such as workshops, courses, and programs outside school settings (Perry & Booth, 2024). Furthermore, facilitators can be actors who study, design, and/or implement a PDP (Lange & Meaney, 2013; Perry & Boylan, 2018). In some cases, more than one facilitator is involved in the PDP, distributing the teaching tasks (Santagata, 2009). Given the relevance of these professionals in PDPs, interest in studying the multiple tasks that they perform in these settings has steadily grown (Lange & Meaney, 2013).

Multiple facilitator-led PDPs have been designed to help mathematics teachers develop their noticing of a specific topic (Jacobs et al., 2024). In this study, we are especially interested in the PDPs focused on mathematics teacher noticing (Fernández & Choy, 2020; Jacobs et al., 2010; van Es & Sherin, 2008). Mathematics teacher noticing is a competence that involves using mathematical and pedagogic knowledge to identify mathematics teaching, interpretation, and decision-making situations (Llinares et al., 2019). In the last 20 years, noticing has gained relevance as it has proven to be an essential teaching practice that can be developed over time (Dindyal et al., 2021; König et al., 2022).

Researchers have mainly examined how teachers notice aspects of student mathematical thinking as they engage in specific tasks such as learning trajectories (Ivars et al., 2018), representations (Friesen & Kuntze, 2021), and problem solving (Jacobs et al., 2010). Research on noticing has also

targeted teaching situations (van Es & Sherin, 2008), educational leaders (Lesseig et al., 2017), classroom management (Gold et al., 2021), sociocultural aspects (Yang et al., 2021), and equity (Kalinec-Craig, 2017; Mendez & van Es, 2024; Warshauer et al., 2023). Other topics that have greatly attracted scholarly attention are teacher knowledge during discussions in the mathematics classroom (Stovall et al., 2024; Superfine et al., 2019; Tekin-Sitrava et al., 2024; van Es & Sherin, 2021) and how teachers orchestrate classroom discussion. In this context, the present study aims to shed light on the facilitation strategies provided by a facilitator during a PDP designed to support mathematics teachers in identifying and interpreting critical events (relevant moments for communicative orchestration seen in a video) in communicative orchestration based on the available evidence. A literature review of recent developments about teacher noticing in mathematics education published between 2019 and 2022 revealed that researchers have rarely examined the relation between noticing and the facilitator (Weyers et al., 2023). The authors highlighted the importance of continuously examining this complex concept across various contexts and populations to gain a deeper understanding of teacher noticing and how it impacts effective teaching and learning. Despite the importance of these topics, studies on mathematics teacher noticing and facilitators have generally focused on how educational leaders notice student mathematical thinking (Amador, 2020; Lesseig et al., 2017), with only a few researchers examining the facilitation strategies employed by PDP facilitators to help teachers guide discussions with their students (González & Skultety, 2018; van Es et al., 2014). The present study focuses on the facilitation strategies provided by a facilitator during a PDP designed to support mathematics teacher noticing of communicative orchestration in the classroom to promote argumentation. The concept of communicative orchestration refers to teaching actions performed by the teacher while promoting argumentation through three communicative approaches: orchestrating participation opportunities, orchestrating through questions, and mitigating errors (Solar & Deulofeu, 2016).

Most research on facilitation and noticing has originated in the United States and central Europe, making it difficult to understand globally and more fully teacher noticing and its implications for the mathematics teaching and learning process. The facilitation strategies used by a facilitator leading a PDP on communicative orchestration in Latin America can yield a relevant contribution to the field and are thus worth studying. We posed the following research question: *What are the characteristics of the facilitation strategies implemented by a facilitator during a PDP designed to support mathematics teacher noticing of communicative orchestration?*

1 Theoretical framework

1.1 Facilitators and PDPs

PDPs, led by a facilitator, support mathematics teachers' learning after the completion of their teacher training (Borko, 2004; Desimone, 2009). Like a teacher in the classroom, a PDP facilitator must lead the discussions of the participating teachers with a view to attaining the program's objectives. This has increased scholarly interest in the role of facilitators (Borko et al., 2014; Coles, 2013). Linder (2011) identified five qualities of an influential facilitator in a mathematics PDP: credibility, support, motivation, management, and personality. Perry and Booth (2024) identified three fields of practice covered by facilitators: content, pedagogy, and corporeality. To shed light on the role of facilitators, Park et al. (2013) examined three elements of a PDP: the aspects that facilitators must pay attention to, the facilitation of discourse aimed at supporting knowledge construction, and the intellectual quality of instructional support. They found that facilitators were more likely to identify matters related to information exchange, with program design and organization issues being less frequently detected. These studies of facilitators account for the diverse qualities of facilitators: they are required to have specific professional characteristics, to support teachers, and to model a process of continuous reflection during their professional practice.

These PDPs may involve the use of a range of resources that shape the program in combination with the objectives to be met (Dobie & Anderson, 2015). For example, videos have made it possible to a) observe the classroom in a realistic and representative form, b) show participants how other teachers work, c) present other styles of classroom teaching, d) suggest ways of analyzing one's practice in order to reflect on teaching, and e) spark debates about the identification of important issues and offer chances to work on these issues in the classroom (Jaworski, 2006). Van Es et al. (2014) examined in-the-moment facilitating moves made by a facilitator in a video-based PDP aimed at offering a framework for facilitation, identifying five moves: a) orienting the group toward the video analysis task, b) maintaining an inquiry-oriented perspective, c) keeping attention on the video and its mathematical contents, and d) supporting collaboration within the group. Their results indicated that the facilitator is essential to the functioning and development of PDPs, and that the roles adopted by the facilitator and the strategies used during the PDP help participants progress in the topic that they are learning about. In the present study, we focus on the *facilitation strategies* applied by the facilitator of one PDP who designed/adapted the program's sessions, implemented these sessions, and served as lead researcher.

1.2 Facilitation in the context of PDPs based on mathematics teacher noticing

In the noticing field, it is common to implement PDPs guided by a facilitator who is tasked with fostering the act of noticing. A variety of PDPs exist to help mathematics teachers develop their noticing competence. Some are brief, limited to a specific portion of the school year (König et al., 2014; Yang et al., 2019); others are implemented over a longer period (Jacobs et al., 2010; Melhuish et al., 2020; Tekin-Sitrava et al., 2024). PDPs in which videos are used to help teachers enrich their professional perspective (Santagata et al., 2021) can be divided into two groups: video clubs, in which teachers analyze their own practice and/or that of other teachers, identifying and interpreting teaching and learning situations in the classroom (van Es & Sherin, 2006; Walkoe et al., 2020; Wallin & Amador, 2019) and PDPs aimed at developing teacher noticing of a specific element (Copur-Gencturk & Tolar, 2022; Sánchez-Matamoros et al., 2015). In all these cases, participating teachers have enriched their noticing of the aspect covered (Lesseig et al., 2017; Wager, 2014). The present study belongs to the second group, since it is about a PDP in which teachers learned about argumentation and communication by identifying and interpreting critical events in the communicative orchestration that they observed in videos.

Existing studies on facilitation and noticing have focused on students' mathematical thinking, particularly through the use of videos in development programs (Santagata et al., 2021). For example, Lesseig et al. (2017) recruited educational leaders who had *noticed* in three ways: a) considering the mathematical work of the facilitator and the teachers featured in video cases; b) linking mathematical work to teachers' learning objectives; and c) reasoning about the facilitator's work to support the development of those learning objectives. Van Es (2009) explored the roles adopted by teachers enrolled in a PDP during discussions about student's mathematical thinking. In that study, the teachers analyzed two videos per session, and the roles adopted by teachers in these discussions were identified. Table 1 shows the roles adopted by the teachers in van Es (2009); these roles inspired the selection and adaptation of the facilitation strategies used by the facilitator in our study.

Table 1

Types of Roles Participants Played in the Video Club Meetings (van Es, 2009)

Type	Role	Definition
Organizational	Coordinator	Informs group what clips will be viewed and presents agenda for meeting
	Clarifier	Ensures all participants have access to same information
	Mediator	Diffuses tension among group members or keeps climate of meeting comfortable
Discussion	Prompter	Invites participants to discuss video clip and initiates issues to explore
	Proposer	Offers issue, explanation, or claim to discuss
	Supporter	Supports a participant's interpretation
	Builder	Challenges claim or explanation
	Critic	Provides additional information to develop an idea
	Summarizer	Synthesizes ideas under discussion
	Blocker	Disregards issues raised and closes the discussion

The different focus of noticing emphasizes how the roles adopted by those involved in a PDP (facilitator or teachers) can influence the ways they are noticed, fostering their participation in discussions about students' mathematical thinking and in identifying, interpreting, and deciding.

PDPs have been shown to be effective in improving the noticing competence of the mathematics teachers who take part in them (Jacobs et al., 2010); however, it is still difficult to identify critical events in the classroom and interpret them using mathematics teaching and learning and the available evidence (van Es & Sherin, 2021). Furthermore, research indicates that teachers find it hard to interpret critical events (van Es & Sherin, 2008, 2010) and justify their interpretations based on video evidence (Jacobs et al., 2024). In our previous study (Arriagada, 2024), we asked five teachers to identify and interpret teaching actions associated with communicative orchestration during a seven-session PDP (three months). We used a rubric to classify their answers into levels and concluded that they achieved higher levels of identification than of interpretation regarding communicative orchestration. Furthermore, we concluded that teachers attain higher levels when they discuss their answers with a group of peers. Therefore, in this study, we explore facilitation strategies implemented by a facilitator during PDP sessions that may have had an influence on the levels of noticing reached by the participants.

1.2.1 Facilitation strategies in the noticing of communicative orchestration

Several studies have shown that certain communicative teaching actions can foster debate in the mathematics classroom (Boerst et al., 2011; Lee, 2006; Solar & Deulofeu, 2016). In a prior study (Arriagada et al., 2025), through

three communicative approaches, we conceptualized noticing of communicative orchestration as a professional competence of mathematics teachers that enables them to i) identify critical events in the teacher's orchestration of the mathematics classroom, particularly regarding communicative teaching actions linked to orchestrating participation opportunities (OP), orchestrating through questions (OQ), and mitigating errors (ME), and ii) interpret the critical events identified in the teacher's orchestration of the mathematics classroom using specific knowledge about communicative teaching actions (Solar & Deulofeu, 2016; van Es & Sherin, 2008).

In this study, we adapt the facilitation strategies provided by a facilitator during a PDP (Zhang et al., 2011). While facilitator-centered facilitation frameworks exist, we selected van Es's teacher-centered framework because it incorporates actions directly related to noticing skills that other frameworks do not. In addition, since an instrument had already been administered to the PDP teacher participants, we needed a facilitation framework aligned with that instrument. To do so, we selected 4 of the 10 roles originally adopted by teachers during PDP discussions (van Es, 2009): prompter, proposer, critic, and builder. We only selected the roles that: i) could be observed in the facilitator's actions, ii) were present in the instrument administered in the previous study during the PDP, and iii) were related to the results of the previous study (identification and interpretation). Thus, we excluded the roles that, in our view, do not make a substantial contribution to evidence-based identification and interpretation of communicative orchestration as approached in our study. We conceptualized the facilitation strategies provided by a facilitator during a PDP as the support that this actor gives teachers for identifying and interpreting critical events in communicative orchestration based on the available evidence (described in the Methodology section, see Table 2). This support consists of verbal actions the facilitator performs while guiding discussions with mathematics teachers during a PDP focused on argumentation and communication. In our opinion, the purpose of the facilitator is not to classify practices as either good or bad (Borko et al., 2011), but to provide tools for the teachers themselves to identify and interpret events, equipping them to discuss the communicative orchestration present in the videos and to exchange views with their peers.

In fact, a recent literature review of research published between 2019 and 2022 revealed that several studies found an association between teacher noticing and professional knowledge; however, it also showed that researchers have rarely examined the relation between noticing and teacher beliefs, between noticing and teaching quality, or between noticing and the facilitator (Weyers et al., 2023). In this context, it is relevant and necessary to study the relation proposed in this study: between the facilitator's role and teacher noticing of communicative orchestration.

2 Methodology

The study has a qualitative design with a case study approach (Yin, 2009). We observed a facilitator's work through actions aimed at helping teachers to develop noticing during the sessions of an argumentation and communication PDP.

2.1 Participants and context

The participants were the facilitator of the PDP and a purposive sample of mathematics teachers who took part in the program (Creswell, 2011). However, for the purposes of the study, we will only analyze and report the facilitator's actions. The facilitator, who served as lead researcher, also adapted and implemented the sessions of the argumentation and communication PDP. The facilitator has eight years of experience as a school teacher (in primary and secondary education), has taught courses on mathematics teaching and learning at undergraduate and graduate levels, has supervised practicums for pre-service mathematics teachers, and has supported mathematics teachers in other PDPs. In addition, the facilitator completed a master's program in case study design for fostering argumentation and a doctoral program on mathematics teacher noticing during a PDP on communication and argumentation. The present study is part of their doctoral thesis (Arriagada, 2024).

The five teachers who took part in the study were contacted using social media. They enrolled voluntarily after confirmation that they met the inclusion criteria: a) being a primary or secondary school mathematics teacher, b) currently teaching or having taught students aged 10–15 years, and c) attending all seven sessions of the argumentation and communication PDP. Of the seven teachers who initially signed up to participate, five completed the PDP (three women and two men). The participants differed in terms of teaching qualifications (primary school with a specialization and secondary school), educational level at which they taught (students from 10 to 18 years old), type of school (public and private), and years of experience (from less than five years to more than 10). Ethical considerations were monitored by the institutional review board, which worked to ensure voluntary participation, anonymization, and data security. The PDP was carried out in 2022.

2.2 The PDP

The argumentation and communication PDP was adapted from the PDP reported in Solar and Deulofeu (2016), based on the Teacher Experience Improvement Model (Solar et al., 2016). It has three stages: analysis of other teachers' practice, design and classroom implementation, and implementation evaluation. It was adapted and implemented by the facilitator to offer tools to promote communication and argumentation in the classroom and improve teacher noticing of communicative orchestration, since this element was

absent from the original design. The program consisted of seven sessions (see Figure 1) involving a variety of resources to assist the participants, including theoretical support material, classroom video analysis, and analysis of tasks present in school textbooks. During the sessions that included video analysis (1, 2, and 3), the facilitator asked the same questions found in the teacher noticing of communicative orchestration instrument (henceforth TNCO) to address argumentation and communication in tandem, via noticing skills. The sequence comprised the following steps: differentiating argumentation from explanation employing videos, emphasizing the analysis of argumentative structure in the classroom; analyzing communicative strategies to foster argumentation through theoretical support and videos; analyzing tasks aimed at promoting argumentation in the classroom found in textbooks; and, finally, designing and implementing a task for promoting argumentation and communication in the classroom.

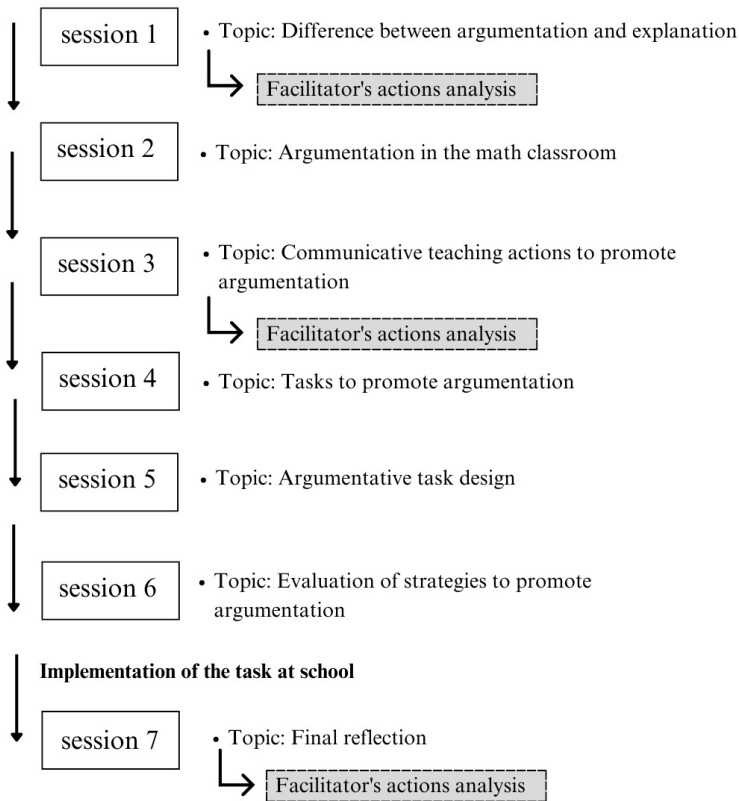


Figure 1
Structure of the PDP

Session 3 is especially relevant for the study, since the facilitator presented the communicative approaches (OP, OQ, and ME), provided the teachers with a list of indicators for each of these approaches (see Appendix 1), and guided the analysis of three videos during the session based on these indicators.

2.3 Administration of the instrument: Background information

The TNCO instrument administered to the participants in the PDP was developed for a prior study (Arriagada et al., 2025), where it was used to characterize teacher noticing. It is composed of three classroom videos (v-1, v-2, and v-3); three questions as a device for promoting noticing (see Appendix 2), one per communicative approach (OP, OQ, and ME); and an analysis rubric comprising four levels, used to identify and interpret communicative actions performed by the teachers in the videos (see Appendix 3). Table 2 provides a summary of the rubric.

Table 2

Summary of the TNCO instrument rubric

Identify	<i>Level 1.</i> The identification of aspects unrelated to the communicative approaches.
	<i>Level 2.</i> The identification of general aspects of the orchestration in connection with the communicative approach.
	<i>Level 3.</i> The identification of critical events in the teacher's orchestration in connection with their communicative actions.
	<i>Level 4.</i> The identification of critical events in the teacher's orchestration with respect to their communicative actions and the relation between these actions and the students.
Interpret	<i>Level 1.</i> The intervention is descriptive and/or evaluative intervention unrelated to the communicative approaches.
	<i>Level 2.</i> The intervention is descriptive and/or evaluative regarding the communicative approach.
	<i>Level 3.</i> The intervention is interpretative regarding the communicative approach, but no evidence is provided.
	<i>Level 4.</i> The intervention is interpretative regarding the communicative approach and includes evidence based on the teacher's actions in the classroom.

The instrument was administered to the teachers in session 1 (v-1), session 4 (v-2), and session 7 (v-3) to capture and characterize their noticing of communicative orchestration. First, the teachers answered individually in writing; then, the facilitator asked them to form groups and share their answers orally. Individual answers and group answers were coded using the analysis rubric for the TNCO instrument for all applications, both for identifying and interpreting.

The results revealed two important aspects. First, across all communicative approaches, group answers reached higher levels than individual answers, both for the identification and the interpretation of critical events in communicative orchestration. Appendix 4 presents examples of individual answers whose level rose after being discussed within a group; this was true of all the participants' individual answers. Table 1 indicates that the individual answers the teachers wrote down were shorter and included no details of the events shown in the video (Level 1 and Level 2). In contrast, the group answers were more detailed and longer; they also incorporated evidence from events found in the video (Level 2, Level 3, and Level 4). For this reason, in this study, we analyzed the facilitator's actions while guiding the discussion with the whole group, as it may have helped to improve the teachers' answers after their small-group discussions. Second, the results showed that, across all communicative approaches, level 4 predominated in identification and level 2 predominated in interpretation. In other words, even though the teachers identified communicative actions that characterized communicative orchestration, they had trouble interpreting those critical events while incorporating evidence from the video. These prior results served as an initial criterion for analyzing the strategies used by the facilitator and determining how their actions might be related to the teachers' noticing.

2.4 Data selection and analysis

The data used in this study were the facilitator's verbal actions aimed at guiding the discussions that took place in the PDP sessions. We selected three sessions: session 1, because it was the first session in which argumentation and communication were discussed; session 3, because the participants analyzed the teacher's communicative actions presented in the TNCO instrument; and session 7, the last session, because the participants engaged in a closing reflection. These three sessions provided the teachers with essential support regarding communicative orchestration. The sessions that were excluded dealt with topics that enriched the discussion on argumentation—e.g., the distinction between argumentation and explanation, the analysis of argumentative tasks in school textbooks, and the design of tasks for promoting argumentation—but did not feature the orchestration of communicative teaching actions to a relevant extent.

To analyze the data, sessions 1, 3, and 7 were transcribed and coded by idea unit (Jacobs & Morita, 2002). In each of the ideas covered in the discussion, we coded the oral interventions of the facilitator using the actions proposed by van Es (2009). Thus, each idea contains multiple actions performed by the facilitator and, therefore, different facilitation strategies. Since we wanted to explore the facilitation carried out by the facilitator in connection with the two results described in the previous section (answers improved by group input and low levels of interpretation), we selected four roles that the facilitator may have adopted to influence these observations: prompter, when the facilitator invites the group to identify; proposer, when the facilitator invites the group to interpret; builder, when the facilitator invites the participants to develop ideas and statements based on evidence to support interpretation; and critic, when the facilitator invites the participants to explore alternative explanations and interpretations. These roles and their respective actions were in the rubric we used to analyze the teachers' answers in the previous study. We refer to these four redefined roles as facilitation strategies for noticing communicative orchestration.

The adaptation is described in Table 3. We selected some of the facilitator's questions and interventions, taken from the discussions with teachers during the video analysis of sessions 1, 3, and 7, to exemplify the actions performed to support evidence-based identification and interpretation. It should be noted that neither the facilitator nor the team were aware of the facilitation strategies beforehand; rather, they were intuitively employed and matched to categories (van Es, 2009).

Table 3

Facilitation strategies for noticing communicative orchestration

Facilitation strategies for noticing communicative orchestration	Actions	Examples of questions asked by the facilitator in this study
Prompter. Invites the group to <u>identify</u> and discuss communicative orchestration (OP, OQ, ME) in the videos.	Solicit. Invites the participants to debate a critical event.	<i>What parts of it caught your attention?</i>
	Redirect. Re-orientes the discussion based on the participants' answers.	<i>Okay, that last part was an explanation [repeats the teacher's answer]. What part of that episode do you think was not an explanation?</i>
	Expand. Invites the participants to develop the idea being debated.	<i>That's still an explanation... Okay, why?</i>

<p>Proposer. Invites the participants to offer explanations about the events identified and substantial interpretations of the communicative orchestration (OP, OQ, ME) in the videos, instead of issuing value judgments.</p>	<p>Judge. Invites the participants to make an assertion of an evaluative nature.</p>	<p><i>What do you think about that? Managing error with a focus on incorrect explanations, not incorrect answers. I mean, trying to find the part of the procedure where the basic error lies, focusing on what is wrong instead of working upon the basis of previously solved problems.</i></p>
	<p>Observe. Invites the participants to see an event in the video segment.</p>	<p><i>What happens in this episode when someone intervenes and says "There's something I don't understand"? When the boy says that, what happens in that moment? What is different about it?</i></p>
	<p>Prescribe. Encourages the participants to advance a course of action that an individual should follow.</p>	<p><i>Does there necessarily have to be another person advancing a counterpoint and answering so that these other elements can come out and generate an enjoyable discussion?</i></p>
	<p>Reason. Invites the participants to make a statement of an interpretative nature.</p>	<p><i>So my question is, is that student advancing a different position or is he asking the girl a question about something he doesn't understand?</i></p>
<p>Builder. Invites the participants to develop ideas and statements based on evidence. Helps the participants to focus on the details of the communicative orchestration; in particular, helps them to use the video and the transcripts as <u>proof of their assertions</u>.</p>	<p>Tag. Invites the participants to offer information tangentially related to the topic being discussed.</p>	<p>Not found</p>
	<p>Connect. Invites the participants to offer details and information directly related to the topic being debated.</p>	<p><i>So, what was the student's idea?</i></p>
<p>Critic. Invites the participants to explore <u>alternative explanations and interpretations</u> of the communicative orchestration events that they have observed (OP, OQ, ME). Generates situations in which the teachers would disagree, challenge, and foster the exploration of alternative explanations about the communicative orchestration observed.</p>	<p>Counter. Invites the participants to challenge the previous speaker's statements.</p>	<p><i>So, what would you say is the difference between explanatory and argumentative? Because Cristofer is unconvicted, he says both are explanations.</i></p>
	<p>Question. Invites the participants to provide an alternative perspective regarding an idea being debated.</p>	<p><i>For what other reason would you say it's interesting, for example, that the teacher calls two students to show what they're doing at the same time?</i></p>

Note. Adapted from van Es (2009). Note. OP=Orchestrating participation opportunities; OQ=Orchestrating through questions; ME=Mitigating errors

We communicatively validated the above facilitation strategies and actions: 30% of the facilitator's interventions were coded by the lead author and discussed with the two coauthors. After resolving the discrepancies that arose, the lead author coded the remaining data.

3 Results

This section is divided into two sub-sections. The first characterizes the facilitation strategies for noticing communicative orchestration adopted by the facilitator through actions observed during the sessions. The second details the actions performed by the facilitator during session 3, when communicative orchestration was first presented to the teachers participating in the PDP.

3.1 Facilitation strategies for noticing communicative orchestration adopted during PDP sessions

During the sessions analyzed, the facilitator implemented multiple facilitation strategies through actions. In session 1, 60% of the facilitator's actions belonged to the prompter facilitation strategy, mainly promoting the identification of aspects that characterize communicative orchestration. In session 3, 54% of the facilitator's actions belonged to the proposer facilitation strategy, inviting the teachers to interpret the communicative orchestration observed in the videos shown during the session. In session 7, 96% of the facilitator's actions belonged to the prompter facilitation strategy, inviting the teachers to identify aspects of the communicative orchestration present in the implementation. The builder and critic facilitation strategies, despite being present, did not predominate in any of the sessions of the PDP. These percentages make sense when we analyze the session-by-session distribution of facilitation strategies, as the number of actions performed in each session differs. For example, in session 1, the prompter facilitation strategy reached 60%, with 12 actions being performed by the facilitator. In contrast, in session 3, this strategy only reached 27%, with 26 actions performed by the facilitator. Table 4 indicates the distribution of the facilitator's actions during the selected sessions.

Table 4
Frequency of facilitation strategies and actions in PDP sessions

Facilitation strategies for noticing communicative orchestration	Actions	Session 1 (Conceptualizing argumentation)	Session 3 (Communicative teaching actions)	Session 7 (Final reflection)
Prompter	Solicit	25% (5)	6.2% (6)	28% (7)
	Redirect	10% (2)	5.2% (5)	20% (5)
	Expand	25% (5)	5.6% (15)	48% (12)
	Prompter total	60%	27%	96%
Proposer	Judge	0%	1% (1)	0%
	Observe	5% (1)	37.5% (36)	0%
	Prescribe	0%	6.2% (6)	0%
	Reason	5% (1)	9.4% (9)	0%
	Proposer total	10%	54.1%	0%
Builder	Tag	0%	0%	0%
	Connect	5% (1)	12.5% (12)	0%
	Builder total	5%	12.5%	0%
Critic	Counter	20% (4)	1% (1)	0%
	Question	5% (1)	5.2% (5)	4% (1)
	Critic total	25%	6.2%	4%
Total		100% (20)	100% (96)	100% (25)

3.1.1 Facilitator as prompter

The first facilitation strategy is prompter. For teachers to develop their identifying skills, they first need to understand what they are asked to identify in the PDP. The facilitator provided theoretical lenses (Appendix 1) that the literature has shown to be effective in improving noticing (Fernández & Choy, 2020; Ivars et al., 2020). With these lenses, the facilitator invited the teachers to identify critical events associated with communicative orchestration in classroom videos and discuss these elements with the whole group. One of the associated actions was to solicit the participants to debate an aspect that they deemed to be critical. For example, this occurred when the facilitator provided a theoretical document with communicative teaching actions classified as orchestrating participation opportunities, orchestrating through questions, and mitigating errors, and then said, “Okay, what did you think of the text? What parts of it caught your attention?” This example shows that the facilitator asked the participants to comment on the teacher actions associated with communicative orchestration present in the text before watching them enacted in the videos. Another action was redirecting the

discussion by mentioning another critical event. For instance, in session 1, while the teachers decided whether the episode observed was an example of argumentation or explanation, one participant stated that the last part of the video was an explanation. In response to this, the facilitator said, “Okay, that last part was an explanation. What part of that episode do you think was not an explanation?” In this example, since the teacher classified the final part as explanatory, the facilitator redirected the discussion by asking about the part that was not explanatory and therefore argumentative. By doing this, the facilitator helped to focus the teachers’ attention on both the explanation and the argumentation. Another action was expanding, as the facilitator encouraged the participants to develop the idea being discussed. The teachers often answered briefly, making it impossible to appreciate the critical event identified in connection with communicative orchestration; therefore, the facilitator asked questions to elicit more in-depth answers. For instance, in the final reflection of session 7, as the participants discussed the implementation shown in the video, a teacher stated that it had been difficult to identify error mitigation actions. In response to this, the facilitator asked, “In what way was it difficult?”, which was an invitation for the teacher to answer in more detail.

3.1.2 Facilitator as proposer

Once the teachers have identified critical aspects connected with communicative orchestration, it is important for them to interpret teacher actions to develop their noticing skill. In the sessions analyzed, the proposer facilitation strategy was used to invite the teachers to offer interpretations and explanations of events already identified in the communicative orchestration present in the videos. The first action was to judge, in which the facilitator encouraged the participants to make an evaluative statement. This happened only once, in session 3, when the facilitator said, “What do you think about that? Managing error with a focus on incorrect explanations, not incorrect answers. That is, trying to find the part of the procedure where the basic error lies, focusing on what is wrong instead of working upon the basis of previously solved problems.” This intervention by the facilitator was an invitation to appraise this way of mitigating errors, prompting the teachers to state whether they thought it was right or wrong. The second action was to observe, in which the facilitator invited the participants to focus on a crucial event by mentioning a communicative action performed by the teacher and asking whether it was present in the episode. For example, the facilitator directed the participants’ attention to a specific communicative action in the video by saying “Let’s look at participation opportunities, including questions in the activity that foster the description and explanation of procedures and ideas. What would you say? Is that present here or not?” This action encouraged

the teachers to justify their answers, providing opportunities to interpret and support the aspect identified using evidence from the video. The second action (observe), was almost exclusively found in session 3, during which the participants analyzed all three communicative approaches, and the communicative actions associated with each approach, one by one. The third action was to prescribe, in which the facilitator encouraged the participants to advance a course of action. This action was only present in session 3, exemplified by interventions such as, “And that’s where we must consider what Isabel said, pay attention not only to the answer, but to the source of the error.” In this passage, based on an answer by a participant, the facilitator indicated an action that a teacher should follow to interpret the student’s error. The fourth action, reason, is crucial for developing the noticing competence. When deploying reason, the facilitator encouraged the participants to offer interpretations of aspects of the communicative orchestration identified in the video. For example, in session 3, the facilitator said, “In a group, the student explains and the teacher in the video says, this is how it works with 1 and 2. So, the teacher says, ‘does it work the same with 3?’... Why do you think the teacher asked that?” This action invited the participants to interpret the orchestration led by the teacher in the video by asking a question, focusing on the communicative actions classified as orchestrating through questions.

3.1.3 Facilitator as builder

By adopting the builder facilitation strategy, the facilitator invited the participants to develop their ideas using the video as evidence. This strategy is essential for developing the noticing competence, since the literature has shown that evidence is fundamental for supporting interpretations (Jacobs et al., 2010; Llinares et al., 2019; van Es et al., 2017). Unfortunately, this strategy was the least commonly adopted by the facilitator. The tag action consists of encouraging participants to share information tangentially related to the topic being debated. Even though we considered this action in our coding work, we did not identify any facilitator actions that reflected its presence. The connect action encouraged participants to link the discussion with details of the communicative orchestration. For example, in session 7, as the whole group discussed the types of questions asked in an argumentative task, the facilitator said, “For example, for you, what would be a good question in that context?” This question invited the participants to pose a question that considered what they had learned during the program, beyond the requirement of establishing whether certain video-recorded actions were appropriate.

3.1.4 Facilitator as critic

The critic facilitation strategy is also crucial for developing teachers' noticing competence. Implementing critic facilitation strategy, the facilitator invited the participants to explore alternative explanations and interpretations of aspects of the communicative orchestration identified in the video. The counter action encourages participants to challenge a previous speaker's comment. For example, in the first session, the teachers watched a video and then had to adopt a position stating whether the episode observed was argumentative or explanatory. Since several different answers were provided, the facilitator said, "I'm going to play you a part [of the video], you're going to check it and then tell me what you had answered initially and if you'd like to change your answer. What would you say about the first episode, the one about the frogs? Is it explanation or argumentation?" This action encouraged the participants to challenge their own answers and/or those of their peers, leading them to focus their attention and interpretation on a specific passage of the video. The second action was to question, as the facilitator invited the participants to advance multiple perspectives regarding the idea under discussion. For instance, in session 3, as the teachers discussed the participation opportunities offered in a video, the facilitator said, "For what other reason would you say it's interesting, for example, that the teacher calls two students to show what they're doing at the same time?" Apart from showing appreciation for the answer provided, this action invited the participants to state another reason why the event in the video was important.

3.2 Facilitation strategies during session 3: Discussions on communicative orchestration

Session 3 is relevant to the study because the teachers who participated in the PDP, under the facilitator's guidance, identified, discussed, and analyzed the communicative teaching actions present in three classroom videos. In prior sessions, the participants differentiated explanation from argumentation; in session 3, they read and identified in videos several communicative actions that could support their argumentation. Throughout session 3, which lasted two full hours, the facilitator performed multiple facilitation strategies, manifested through actions presented in Table 5.

Table 5
Facilitation strategies and actions of the facilitator during session 3

Idea n°	Topic of the idea	Main facilitation strategy	Main actions
1	Read and comment on a text	Prompter	Solicit-Redirect
2	Discuss participation opportunities in video 1	Prompter	Redirect
3	Discuss error mitigation in video 1	Prompter	Solicit
4	Discuss question orchestration in video 1	Prompter	Solicit
5	Discuss participation opportunities in video 2	Proposer	Observe
6	Discuss error mitigation in video 2	Proposer Builder	Observe Connect
7	Discuss question orchestration in video 2	Proposer	Observe
8	Discuss overall communicative orchestration in video 3	Proposer Builder	Observe-Reason Connect

In this session, the teachers were equipped with a document that included the list of teacher actions found in communicative orchestration proposed by Solar and Deulofeu (2016), plus a table with the same actions they could use to record whether these actions were present. While discussing idea 1, the participants commented on the communicative actions in the document. During the whole group discussion, the facilitator tended to employ the prompter facilitation strategy, mainly asking the teachers to identify crucial events in the document. During the discussion of ideas 2, 3, and 4, the facilitator operated as a prompter, generally requesting that the participants identify the communicative actions from the document in the video, considering the orchestration of participation opportunities, error mitigation, and orchestration through questions. However, during the discussion of video 2, ideas 5, 6, and 7, there was a change in the facilitator's actions, marked by the predominance of the proposer strategy and to a lesser extent the builder strategy. For example, during the discussion of the three aspects of communicative orchestration (OP, OQ, ME), the facilitator acted as a proposer, encouraging the participants to observe specific actions in the videos. While discussing error mitigation in video 2, the facilitator was a builder, inviting the participants to make connections by providing details related to the topic being debated. Finally, in idea 8, while discussing video 3, which the teachers had watched in a prior session, the facilitator acted as a proposer and a builder at the same time. The facilitator was a proposer because they invited the participants to watch specific parts of the video as

a means of discussing the students' problems, the teacher's orchestration to avoid providing answers, and other aspects of the class. This idea also featured actions associated with reasoning, nearly absent from session 1, as the facilitator encouraged the participants to interpret the teacher's action in video 3. Lastly, the facilitator adopted the role of a builder through the action connect, inviting the teachers to provide further details. In all cases, this came after a reasoning or observing action; that is, after the teachers had identified and interpreted a crucial event in the communicative orchestration.

4 Discussion and conclusion

Our research question was: What are the characteristics of the facilitation strategies implemented by a facilitator during a PDP designed to support mathematics teacher noticing of communicative orchestration?

We know that the facilitators of a PDP play a key role in supporting teachers: designing the PDP, inviting teachers to participate, implementing the PDP, and evaluating the PDP and/or conducting research based on it (Lange & Meaney, 2013; Nieman et al., 2023; Ricart et al., 2022). However, in terms of mathematics teacher noticing (Fernández & Choy, 2020; Jacobs et al., 2010; van Es & Sherin, 2008), we know little about facilitation strategies and even less about experiences with a focus on teachers' noticing of communicative orchestration. In this context, adapting the framework of van Es (2009), originally designed for teaching actions, allowed us to focus the analysis on the specific facilitation strategies of the teacher noticing skill. Our study allowed us to analyze relevant aspects of the facilitator, such as how the facilitator requests the identification of critical events in communicative orchestration (typical of the prompter facilitation strategy) or how the facilitator requests a focus on evidence (typical of the builder facilitation strategy). Our findings indicate that the facilitator mainly promoted the facilitation strategies of prompter and proposer, inviting teachers to identify and interpret critical events in the communicative orchestration.

The facilitator was mainly a prompter, requesting that the teachers identify critical events of the communicative orchestration in the classroom videos shown during the PDP and encouraging them to expand their answers and explanations of the events observed. This result is consistent with the design of the PDP, since the teaching actions (Solar & Deulofeu, 2016) analyzed in session 3 were unknown to the participants; therefore, it was necessary to equip them for identifying such actions. Even though the prompter facilitation strategy predominated during sessions 1 and 7 in terms of percentage, the facilitator performed a larger number of actions in session 3. The facilitator also adopted a proposer role, chiefly in session 3, encouraging the teachers

to determine whether the specificity indicators discussed were present in the videos used in the session and inviting them to reflect on these interpretations. It is relevant to note that the facilitator almost never employed judge actions. We consider that this is a positive result, because encouraging participants to become judges means inviting them to pass judgment on the performance of colleagues without considering contextual characteristics, and especially without the protective and respectful space that PDP sessions afford. Other studies have shown that teachers tend to judge or make rushed appraisals of what they observe (van Es & Sherin, 2008, 2010), failing to explain how they have reached their conclusions; therefore, promoting evidence-based interpretation can enrich teacher discussion and reflection on teaching practice.

In general, we appreciate the fact that the prompter and proposer facilitation strategies, through their respective actions, were present in the sessions analyzed. However, we would have expected the reason action, part of the proposer facilitation strategy, to feature more prominently in the facilitator's actions, since this is the action that encourages teachers to interpret the communicative orchestration of the teacher in the videos. This finding also poses a challenge for future interventions, since our previous study showed that teachers attained high levels of identification of communicative orchestration in all three communicative approaches covered in the rubric (Levels 1, 2, and 4), though not in the interpretations section (Levels 1 and 2). In this regard, we conjectured that, if teachers need more opportunities to develop their noticing of communicative orchestration, it is the facilitator's duty to provide them.

The low presence of the builder and critic facilitation strategies constitutes a challenge for future implementations and for facilitator training. We conjectured that the scarcity of the builder role may have been because transcripts were not used in this PDP. Transcripts enrich discussions and suggest alternative interpretations (Amador et al., 2021), resulting in higher levels of interpretation in the TNCO instrument or other instruments that involve the use of evidence (Jacobs et al., 2024; van Es, 2011). However, it is worth noting that the facilitator performed connect actions 12 times in session 3, inviting the participants to add details based on video evidence. Although we considered the tag action in our analysis of the sessions, we found no instances of the facilitator performing it. This leads us to evaluate whether this action is suited to our adaptation of the instrument or whether the facilitator did not promote it. Within the critic role, it is worth highlighting that the facilitation strategy question was present in the three sessions analyzed, suggesting that it is possible to invite participants to advance new perspectives without necessarily linking these efforts to a specific PDP objective. Nevertheless, fostering the critic role is a challenge that we intend

to address in a future version of the PDP. The teachers linked their interpretations of aspects of communicative orchestration to evidence taken from the video; also, their group responses exhibited high levels of interpretation regarding error mitigation (level 4, interpretation). Therefore, we consider that the facilitator offered opportunities for the teachers to interpret communicative orchestration themselves, using evidence to support their views.

In the previous study, we were surprised to find that the teachers' group answers reached higher levels than individual ones in identification and interpretation. In this regard, like van Es (2009), we consider that attempting to explain ideas to others provides additional evidence that makes it possible to render these ideas more readily understandable, which consequently increases noticing. In that sense, encouraging discussion can enrich the overall quality of the debate and thus improve teachers' noticing of communicative orchestration (van Es et al., 2014; Zhang et al., 2011). Consequently, PDPs will continue to offer opportunities for teachers to enrich their practice.

4.1 What we learned about facilitation strategies

This study yielded several lessons for us as educators and researchers. First, that facilitation strategies are related to the topic of each session. The session with the largest presence of facilitation strategies for the teachers to identify and interpret communicative orchestration was session 3, in which they familiarized themselves with these communicative approaches and analyzed communicative actions through videos.

Second, if the answers of the teachers who take part in a PDP have been analyzed with noticing frameworks, it is important to also employ facilitation frameworks centered on noticing to analyze the facilitator (van Es, 2009). Although some facilitator-centered facilitation frameworks focus on the facilitator (Linder, 2011; Perry & Booth; 2024; van Es et al., 2014;), selecting the framework that includes the skills that typify noticing, such as identification, interpretation, use of evidence, and alternative interpretations, among others, enriches the connection established with the development of teachers' noticing. Although our chosen framework was originally formulated to describe teacher actions (van Es, 2009), our adaptation requires some changes to analyze the facilitator's actions with greater accuracy.

Third, while we appreciate the strategies that facilitated the identification and interpretation of communicative orchestration, we conjectured that facilitation strategies aimed at promoting interpretation still represent a challenge, as reported in previous studies (Jacobs et al., 2010; van Es & Sherin, 2021). Even though the facilitator helped the teachers identify specific critical events in communicative orchestration and argumentation throughout the three sessions, the same was not true of evidence-based interpretation.

In this context, promoting the builder facilitation strategy, consisting of tag actions (absent) and connect actions (low presence), is crucial for teachers to develop an interpretation of what is observed during the discussions.

Fourth, we conjectured that a facilitator's expertise in argumentation and communication on the one hand and in noticing on the other does not necessarily ensure good practices during a PDP, and their efforts are therefore worth analyzing (Amador et al., 2025). It should be noted that the facilitation framework was used to analyze the facilitator's interventions after conducting the PDP and after analyzing teacher responses, so having this framework prior to the PDP could also generate greater consistency between both data sources.

Finally, we would like to emphasize the relevance of analyzing the connections between the facilitator's actions and the development of teacher noticing. In this regard, research conducted mainly in the United States and central Europe shows that this competence develops over the course of several years and multiple PDPs (Jacobs et al., 2024), stressing the importance of determining which facilitation actions can contribute to its growth while also benefiting teachers' overall performance.

4.2 Limitations and projections

Given that the study was conducted after the COVID-19 pandemic (in 2022), we implemented a brief PDP with few participants in consideration of the restrictions still in place during the return to in-person activities. This limited the number of sessions of the PDP (seven); the number of sessions should be increased in a future implementation since noticing improves over time (Jacobs et al., 2024). Furthermore, the low number of participants precluded other types of analysis, such as finding trends in specific sessions or establishing associations between the facilitator's roles and particular sessions. Another limitation worth noting is that the facilitator and the lead researcher were the same person, which may have biased the data analysis process. However, the greatest benefit that this coincidence brought to the study was that it enabled the facilitator to observe their own practice and thus identify opportunities for improvement, since there are few tools for facilitators to examine their practice continuously (Nieman et al., 2023). Even though the facilitator is not always the researcher, in this study they were one and the same; therefore, the perspectives of the other members of the research team were essential for progressively discussing facilitation strategies.

The facilitator of a PDP plays an essential role, because they are responsible for the participants' learning (Borko, 2004). Nevertheless, it is difficult to connect the facilitator's role with the learning of the teachers who participate in a PDP. For this reason, as part of the projections of the study, we hope to continue exploring the facilitation strategies deployed by facilitators in

connection with teachers' answers in a longitudinal study, since this actor is crucial to the development of teachers' noticing of communicative orchestration. The facilitator of a PDP has an essential function to perform, being responsible for leading discussions while negotiating complex and diverse tasks such as program design, planning, and execution. Facilitators are under-researched and thus undervalued as professionals (Perry & Booth, 2024); in this context, our study has relevant implications for those entrusted with supporting mathematics teachers in PDPs.

Acknowledgments

This work was supported by the Chilean National Agency of Research and Development (ANID) under National Doctoral Grant 21201121, ANID Fondecyt 1231303 and the Ph.D. in Education program at Pontificia Universidad Católica de Chile.

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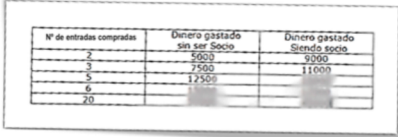



Appendix 1

Communicative teaching actions (Solar & Denlofen, 2016)

Communicative approaches	Description	Communicative teaching actions
Orchestrating participation opportunities (OP)	Teacher action aimed at ensuring that all students get the chance to contribute	<p>OP-1. Walks around the classroom watching student progress, observing multiple procedures, incorrect answers, or frequent errors.</p> <p>OP-2. Promotes the discussion of multiple procedures, questions, or answers used to solve a problem.</p> <p>OP-3. Promotes the participation of all students, not only those who wish to participate.</p> <p>OP-4. Flexibly manages the class so that the students can interrupt the teacher and participate.</p> <p>OP-5. Does not validate or invalidate the questions, answers, or procedures that students used to solve a problem.</p>
Orchestrating through questions (OQ)	Teacher action to formulate questions appropriate to the activity or to interact based on good questions.	<p>OQ-1. Asks questions that favor description and explanation of ideas/procedures over closed questions.</p> <p>OQ-2. Does not ask rhetorical questions (asking the question and answering immediately).</p> <p>OQ-3. Asks counter-questions to students based on the answers given by them.</p> <p>OQ-4. Returns good questions posed by students to the rest of the class.</p> <p>OQ-5. Asks questions that invite reflection and deepening.</p>
Mitigating errors (ME)	Teacher action to convey to students that their ideas, answers, and incorrect procedures are important in constructing mathematical knowledge.	<p>ME-1. Promotes students expanding on correct and incorrect answers, without previously validating the quality of their answers.</p> <p>ME-2. Mitigates error by collectively seeking mathematical knowledge to improve an initial answer.</p> <p>ME-3. Anticipates possible incorrect answers of students related to the mathematical content.</p> <p>ME-4. Promotes assertive and constructive discussion among students about incorrect answers.</p> <p>ME-5. Uses error to resolve questions.</p> <p>ME-6. Mitigates error, focusing on incorrect explanations and not on the correct answers.</p>

Appendix 2

Background information about the videos included in the TNCO instrument and questions posed to the teachers

<p>Video assignment v-1</p> <p>Cinema Paraiso. At Cinema Paraiso, there is the possibility of becoming a member of the cinema by purchasing an annual subscription card, which allows the purchase of movie tickets at a discounted cost per ticket.</p> <p>The following table contains a report of what a person would spend buying movie tickets as a member and as a non-member. Unfortunately, the report has been wetted down and some of the information has been lost.</p>  <p>A family says that by taking the most convenient option their annual savings last year was 29,000 Chilean pesos, how many tickets did they buy?</p>	<p>Video context v-1</p> <p>The video corresponds to class 1, of a sequence of 3 classes, where the teacher implements an activity to promote argumentation and mathematical modeling simultaneously. During this class, students discussed the problem and recorded their resolutions on Kraft paper. In the second class the groups presented their resolutions and in the third class they validated the models found.</p>  <p>Class 1. Group work</p>
<p>Video assignment v-2</p> <p>The 7th grade of a school wants to go on a study tour to the mountains. Company "A" charges them a fixed daily rate of 81,000 Chilean pesos and 120 Chilean pesos for each kilometer traveled, while company "B" has a fixed daily rate of 90,000 Chilean pesos and charges 80 Chilean pesos for each kilometer traveled. If you plan to travel many kilometers, which company's lease is more convenient?</p>	<p>Video context v-2</p> <p>The video corresponds to class 1, of a sequence of 4 classes, where the teacher implements an activity to promote argumentation and mathematical modeling in an articulated way. During this class, the students discussed the problem and most of the groups indicated that the most convenient company was "A", however, when they tried with a larger number of kilometers, they realized that company "B" was more convenient.</p>  <p>Class 1. Group work</p>
<p>Video assignment v-3</p> <p>Join two polyhedra from their base and calculate the perimeter of the new figure.</p>  <p>Class. Large group work</p>	<p>Video context v-3</p> <p>The video corresponds to the editing of a class, where the teacher implements an activity to promote argumentation. Before starting the task, the teacher wants to check if the students recognize the polyhedra they are going to work with. To do so, she presents several figures and asks them to identify the figures with the same name on both sides of the central line.</p>
<p style="text-align: center;">Question asked to participants</p> <ol style="list-style-type: none"> 1. What do you notice about how the teacher orchestrates the opportunities for student participation that she offers? Observation: This refers to the teacher's action in the video to ensure that all students could contribute. 2. What do you notice about how the teacher orchestrates the discussion with her students through questions? Observation: refers to the teacher's action in the video to formulate questions appropriate to the activity or to interact based on good questions. 3. What do you notice about how the teacher mitigates error of her students? Observation: It refers to the action of the teacher in the video to convey to her students that their wrong ideas/answers/procedures are important to build mathematical knowledge. 	

Appendix 3

Instrument rubric Teacher Noticing Argumentative Orchestration

What do teachers identify?			
Level 1	Level 2	Level 3	Level 4
<i>Orchestrating participation opportunities to foster communication (OP)</i>			
Identifies general aspects of the teacher's orchestration in the video that are not related to the OP. For example, classroom climate, motivation, etc.	Identifies general aspects of the teacher's OP, without relating them to any of the indicators.	Identifies critical events of the teacher's OP, in relation to one of the indicators (communicative teaching actions).	Identifies critical events of the teacher's OP in relation to one of the indicators and indicates the effect that the OP carried out by the teacher generates in the students.
<p>Indicators:</p> <p>OP-1. Walks around the classroom watching student progress, observing different procedures, incorrect answers, or frequent errors.</p> <p>OP-2. Promotes the discussion of different procedures, questions, or answers used to solve a problem.</p> <p>OP-3. Promotes the participation of all students, not only those who wish to participate.</p> <p>OP-4. Flexibly manages the class so that the students can interrupt the teacher and participate.</p> <p>OP-5. Does not validate or invalidate the questions, answers, or procedures of students used to solve a problem.</p>			
<i>Orchestrating through questions to foster communication (OQ)</i>			
Identifies general aspects of the teacher's orchestration that are unrelated to the teacher's OQ (e.g., classroom climate, motivation).	Identifies general aspects of the teacher's OQ that are unrelated to the indicators (see level 3 indicators).	Identifies critical events of the teacher's OQ that are related to any of the indicators (communicative teaching actions).	Identifies critical events of the teacher's OQ that are related to any of the indicators (see level 3 indicators) and states the effect of the teacher's OQ on the students.
<p>Indicators:</p> <p>OQ-1. Asks questions that foster the description and explanation of ideas/procedures rather than closed questions.</p> <p>OQ-2. Does not ask rhetorical questions, that is, does not ask a question and answer it immediately.</p> <p>OQ-3. Asks students follow-up questions based on their answers.</p> <p>OQ-4. Restates to the whole class good questions asked by students.</p> <p>OQ-5. Asks questions that encourage reflection and deeper analysis.</p>			

<i>Mitigating errors to foster communication (ME)</i>			
Identifies general aspects of the teacher's orchestration that are unrelated to ME in mathematics. (e.g., classroom climate, motivation).	Identifies general aspects of the teacher's ME that are unrelated to the indicators (see level 3 indicators).	Identifies critical events of the teacher's ME that are related to any of the indicators (communicative teaching actions).	Identifies critical events of the teacher's ME that are related to any of the indicators (see level 3 indicators) and states the effect of the teacher's ME on the students.
	<p>Indicators:</p> <p>ME-1. Promotes students expanding on correct and incorrect answers, without previously validating the quality of their answers.</p> <p>ME-2. Mitigates error by collectively seeking mathematical knowledge to improve the initial answer.</p> <p>ME-3. Anticipates possible incorrect answers of students related to the mathematical content.</p> <p>ME-4. Promotes assertive and constructive discussion among students about incorrect answers.</p> <p>ME-5. Uses errors to solve questions.</p>		
What do teachers identify?			
Level 1	Level 2	Level 3	Level 4
Descriptive and/or evaluative of the classroom in general. Within the interpretation there are generalities or value judgments related to the classroom in general.	Descriptive and/or evaluative. Within the interpretation, there are generalities or value judgments related to the specific indicators (OP, GO, and ME).	Interpretive. Interpretation is related to the communicative teaching actions of a specific indicator (OP, GO, and ME). However, no evidence from the video is included.	Interpretive. Interpretation is related to the communicative teaching actions of a specific indicator (OP, GO, and ME) and includes evidence from the video.

Extracted from Arriagada et al. (2025)

Appendix 4

Teacher answers to TNCO questions

Approach	Answer	Example	Identify	Interpret
OP (v-2)	Individual	I don't get it. I don't know, I always see her with only one student. The students describe how they solve their problems. (Cristofer)	Level 1	Level 1
	Group	Cristofer: Yes, I only saw one of the kids she talked about; she suddenly got up and went to talk with another girl in the group, but even that was weird, because she went to talk to this other girl, but she could have talked through the other desks, she had a diagonal line. The thing that really caught my attention, regarding how she talked with them, is that the kids always described what each of them was doing, she didn't do that herself... instead of reading their solutions and trying to understand, instead of saying "oh, okay, this, this, and that", and then "here I multiplied by three, by five"... I get the impression everyone was telling her that answer E was right.	Level 3	Level 2
OQ (v-1)	Individual	He's trying to make sure everyone or most of the group agree with the group's answer. He wants them to clarify what they're answering and why. (Sandy)	Level 1	Level 1
	Group	Sandy: The fact that they got an explanation of what their classmate had done, that's quite helpful, listening to their classmate; they were like, um, look, what if we do something else? And that's when the discussion starts. I wrote that the teacher first makes sure that everyone or most of the students agree. She said that many times, "Do you agree with what your classmate said?" Or make sure everyone... she said that many times, made sure they all agree with what they did. And the other point was to make sure their answers were clear. Because when they were answering the question, she made them check the question again.	Level 3	Level 2

ME (v-1)	Individual	The questions she asked at the beginning were a way to make the students identify what they're being asked, but she doesn't guide them so they can recognize the questions. When they encounter errors, she asks follow-up questions about the wording of the problem. There are errors in the formulation of the problem. (Isabel)	Level 2	Level 2
	Group	Isabel. Well, I wrote that when they find errors, she asks follow-up questions about the formulation of the problem, they're focusing on the definition of what they're being asked about, and that's because there are mistakes in the wording of the problem. I wrote that the questions she asks at first are a way to help the students to identify what they're being asked about, but she doesn't guide them so they can identify the question, except for the last one... the last ones were progressing better, so she asked them, "Am I asking you about this thing over here or that thing over there?"	Level 4	Level 4